

FERMION MASSES AND MIXING IN SUSY GUT *

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The problem of fermion masses and mixings is discussed in the context of super-symmetric grand unification theories. Some predictive frameworks based on the $SU(5)$, $SO(10)$ and $SU(6)$ models are reviewed.

1. Introduction*1.1. Family Problems*

The problem of fermion flavours (or families) is one of the key problems in modern particle physics. It has different aspects, questioning origin of the family replication (why three families?), quark and lepton mass spectrum and mixing pattern, CP violation in weak interactions, CP conservation in strong interactions, suppression of the flavour changing neutral currents (FCNC), pattern of neutrino masses and oscillations, etc.

The Standard Model¹ (SM) can be considered as a minimal theory of flavour. Being an internally consistent renormalizable gauge theory, it has been extremely successful in describing various experimental data accumulated over the past several years. It is likely that the SM is a literally correct theory at presently available energies. It accommodates all observed quarks and leptons in a consistent way. Three families sharing the same quantum numbers under the $SU(3) \times SU(2) \times U(1)$ gauge symmetry are introduced as an anomaly free set of chiral left-handed (LH) fermions $q_i = (u_i, d_i)$, u_i^c , d_i^c ; $l_i = (\nu_i, e_i)$, e_i^c , where $i = 1, 2, 3$ is a family index (q, l are the weak isodoublets, and the isosinglet states u^c, d^c, e^c are the C-conjugates to the right-handed (RH) components u_R, d_R, e_R). A remarkable feature of the SM is that the fermion and the gauge boson W^\pm, Z masses have a common origin, namely the Higgs mechanism. In fact, fermions would remain massless as far as gauge symmetry is unbroken. They get masses through the Yukawa couplings to the Higgs doublet ϕ :

$$\mathcal{L}_{\text{Yuk}} = \lambda_{ij}^u q_i C u_j^c \tilde{\phi} + \lambda_{ij}^d q_i C d_j^c \phi + \lambda_{ij}^e l_i C e_j^c \phi \quad (\tilde{\phi} = i\tau_2 \phi^*) \quad (1)$$

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So, the fermion masses are related to the weak scale $\langle\phi\rangle = v = 174$ GeV. However, the Yukawa constants remain arbitrary: $\hat{\lambda}^{u,d,e}$ are general complex 3×3 matrices.

The SM contains no renormalizable couplings that could generate the neutrino masses. As far as renormalizable interactions are concerned, the lepton and baryon number conservations arise as accidental symmetries of the theory. The lowest order couplings relevant for the neutrino masses are the $d = 5$ ones²

$$\mathcal{L}_\nu = \frac{\lambda_{ij}^\nu}{M} (l_i \tilde{\phi}) C (l_j \tilde{\phi}), \quad \lambda_{ij}^\nu = \lambda_{ji}^\nu \quad (2)$$

where $M \gg v$ is some regulator scale. In the seesaw picture³ these can be induced by exchange of the heavy RH neutrinos. In this case M is related to the mass scale of the latter. Alternatively, if the SM is valid all the way up to planckian energies (i.e. there are no RH neutrinos with mass $\leq M_{Pl}$), then operators (2) with $M \sim M_{Pl}$ could effectively emerge due to the non-perturbative quantum gravitational effects⁴. In this view, the value $\hat{m} = v^2/M_{Pl} = 3 \cdot 10^{-6}$ eV can be regarded as a natural unit of the neutrino masses in the SM.

The coupling constant matrices and correspondingly the fermion mass matrices $\hat{m}^f = \hat{\lambda}^f v$ ($f = u, d, e$) and $\hat{m}^\nu = \hat{\lambda}^\nu (v^2/M)$ can be brought to the diagonal form by the unitary transformations:

$$V_f \hat{m}^f V_f' = \hat{m}_{diag}^f, \quad V_\nu \hat{m}^\nu V_\nu^T = \hat{m}_{diag}^\nu \quad (3)$$

where

$$\begin{aligned} \hat{m}_{diag}^u &= \text{diag}(m_u, m_c, m_t) = v \cdot \text{diag}(\lambda_u, \lambda_c, \lambda_t) \\ \hat{m}_{diag}^d &= \text{diag}(m_d, m_s, m_b) = v \cdot \text{diag}(\lambda_d, \lambda_s, \lambda_b) \\ \hat{m}_{diag}^e &= \text{diag}(m_e, m_\mu, m_\tau) = v \cdot \text{diag}(\lambda_e, \lambda_\mu, \lambda_\tau) \\ \hat{m}_{diag}^\nu &= \text{diag}(m_1, m_2, m_3) = \frac{v^2}{M} \cdot \text{diag}(\lambda_1, \lambda_2, \lambda_3) \end{aligned} \quad (4)$$

Hence, quarks are mixed in the charged current interactions:

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{(u_1, u_2, u_3)}_L \gamma^\mu W_\mu^+ \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L = \sqrt{2} g \overline{(u, c, t)} \gamma^\mu (1 + \gamma^5) W_\mu^+ V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (5)$$

where $V = V_u^+ V_d$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In a convenient parametrization it has a form

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (6)$$

where c_{ij} and s_{ij} respectively stand for the ‘cos’ and ‘sin’ of the mixing angles θ_{12} , θ_{23} and θ_{13} , and δ is a CP-violating phase. In the case of massive neutrinos, a similar mixing matrix $V_{Lept} = V_\nu^+ V_e$ emerges also in the lepton sector.

The mass spectrum of the quarks and charged leptons is spread over five orders of magnitude, from MeVs to 100 GeVs:⁵:

$$\begin{aligned} m_t &= 170 \pm 12 \text{ GeV}, & m_c &= 1.3 \pm 0.1 \text{ GeV}, & m_u &= 2 - 8 \text{ MeV} \\ m_b &= 4.3 \pm 0.2 \text{ GeV}, & m_s &= 100 - 300 \text{ MeV}, & m_d &= 5 - 15 \text{ MeV} \\ m_\tau &= 1.784 \text{ GeV}, & m_\mu &= 105.6 \text{ MeV}, & m_e &= 0.511 \text{ MeV} \end{aligned} \quad (7)$$

Following the tradition⁶, for the heavy quarks t, b, c , we refer to their running masses respectively at $\mu = m_{t,b,c}$, and for the light quarks u, d, s – at $\mu = 1 \text{ GeV}$. For the top quark ‘pole’ mass $M_t = m_t[1 + (4/3\pi)\alpha_3(m_t)]$ the recent results of the CDF and D0 groups⁷ imply respectively $M_t = 176 \pm 8 \pm 10 \text{ GeV}$ and $M_t = 199 \pm 20 \pm 22 \text{ GeV}$, with the average $M_t = 180 \pm 12 \text{ GeV}$. This is in a good agreement with the present precision data on the SM.

The light quark masses are the less known quantities in (7), however their ratios are known with the better accuracy⁸:

$$\frac{m_u}{m_d} = 0.25 - 0.70, \quad \frac{m_s}{m_d} = 17 - 25; \quad \left(\frac{m_u}{m_d}\right)^2 + \frac{1}{Q^2} \left(\frac{m_s}{m_d}\right)^2 = 1 \quad (Q = 23 \pm 2) \quad (8)$$

The weak transitions dominantly occur inside the families, and are suppressed between different families. For the CKM mixing angles we have⁵

$$\begin{aligned} |V_{us}| &= s_{12} = 0.222 \pm 0.002 \\ |V_{cb}| &= s_{23} = 0.040 \pm 0.005 \\ |V_{ub}| &= s_{13} = (0.08 \pm 0.02) \cdot s_{23} \end{aligned} \quad (9)$$

Direct measurements show no evidence for any of the neutrinos to be massive, providing only the upper bounds⁵:

$$m_{\nu_\tau} < 31 \text{ MeV}, \quad m_{\nu_\mu} < 270 \text{ keV}, \quad m_{\nu_e} < 7.0 \text{ eV} \quad [2\beta_{0\nu} : m_{ee}^\nu < 0.7 \text{ eV}] \quad (10)$$

On the other hand, there have been indirect “positive” signals for neutrino masses and mixing accumulated during the past years. The most serious hint among these is related to the *solar neutrino problem* (SNP). A solar neutrino deficit indicated by the current experimental data cannot be explained by nuclear/astrophysical reasons⁹. This points that the SNP is rather due to the neutrino properties, the most natural and plausible solution being the solar ν_e oscillation into another neutrino ν_x ($\nu_x = \nu_\mu$ or ν_τ). It can explain the experimental data in two following regimes.

*Just-so*¹⁰: long wavelength oscillation from Sun to Earth, with parameters in the range

$$\delta m_{ex}^2 \sim 10^{-10} \text{ eV}^2, \quad \sin^2 2\theta_{ex} \sim 1 \quad (11)$$

i.e. neutrino masses $\sim \hat{m}$ and almost maximal mixing. In the context of the operators (2) this implies that $M \sim M_{Pl}$ and all $\lambda_{ij}^\nu \sim 1$, as it could emerge from the Planck scale physics⁴.

MSW^{11} : resonant oscillation inside the solar medium, with the parameter range

$$\delta m_{ex}^2 \sim 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{ex} \sim 10^{-2} \quad (12)$$

This case favours smaller scale $M \sim 10^{12-16}$ GeV, and the “hierarchical” form of the matrix λ_{ij}^ν aligned to the charged leptons Yukawa matrix λ_{ij}^e .

Other hints, as are the atmospheric neutrino deficit¹², LSND anomaly¹³ or the “after COBE” evidence for some hot fraction of the cosmological dark matter¹⁴, point to heavier ($m_\nu \sim 0.1 - 10$ eV) and substantially mixed neutrinos.

Summarizing, one can conclude that the SM accomodates the fermion sector in a consistent way. There is only one dimensional parameter, $v = 174$ GeV, which determines the mass scale of the charged fermions. It is of key importance that the SM exhibits the *natural* suppression of the flavour changing neutral currents (FCNC), both in the gauge boson and Higgs exchanges¹⁵. Since the Yukawa constants in (1) are generally complex, the observed CP-violating phenomena can be explained by the CKM mechanism with sufficiently large CP-phase ($\delta \sim 1$). However, at the same time this creates the strong CP problem¹⁷: the overall phase of the complex Yukawa matrices would effectively contribute to the Θ -term in QCD and thus induce the P and CP violation in strong interactions. On the other hand, absence of the dipole electric moment of neutron puts a strong bound $\Theta < 10^{-9}$.

1.2. Fermion masses beyond the Standard Model

As noted above, the fermion mass and mixing problem can be phrased as a problem of the Yukawa coupling matrices $\hat{\lambda}^f$ which remain arbitrary in the SM: there is no explanation, what is the origin of a strong hierarchy between their eigenvalues, why $\hat{\lambda}^u$ and $\hat{\lambda}^d$ are aligned so that the CKM mixing angles are small, what is the origin of the complex structure needed for the CP-violation in weak interactions, why the Θ -term is vanishingly small in spite of the complex Yukawas, etc.

It is attractive to think that at some intermediate scale M_F between the electroweak and Planck scales there exists a more fundamental theory which could allow to calculate the Yukawa couplings, or at least somehow constrain them.

In order to analyze predictions of such a theory, one has to compare the quark and lepton running masses at the scale $\mu \sim M_F$. The latter are related to the ‘physical’ masses (7) through the renormalization group (RG) equations. One has to remember, however, that these equations contain a principally unknown parameter:

$$\mathcal{X}(M_F) = \text{Spectrum of particles below the scale } M_F \quad (13)$$

The minimal assumption is that $\mathcal{X}(M_F) = \mathcal{X}_{\text{SM}}$, i.e. below the scale M_F the theory reduces to the minimal SM literally in all its sectors (gauge, fermion, Higgs) and there are no extra particles besides the known ones. (In the context of the MSSM there will be also their superpartners with masses $m_S \sim 1$ TeV). In this case the RG running is essentially defined by the gauge coupling constants g_3, g_2, g_1 of the $SU(3) \times SU(2) \times U(1)$ symmetry and large top Yukawa constant ($\lambda_t \sim 1$). The

effects of large λ_t imply the existence of the infrared-fixed point¹⁸ and also influence the RG running of other Yukawa constants and mixing angles¹⁹.

Looking on the fermion mass spectrum at the scale $\mu \sim M_F \gg M_W$, we observe that it is divided into following groups (in units of the weak scale $v = 174$ GeV):

$$m_t \sim v, \quad m_{c,b,\tau} \sim 10^{-2}v, \quad m_{s,\mu} \sim 10^{-3}v, \quad m_{u,d,e} \sim 10^{-5}v \quad (14)$$

One can also observe that the *vertical* mass splitting is small within the first family of quarks and is quickly growing with the family number:

$$\frac{m_u}{m_d} \sim 1, \quad \frac{m_c}{m_s} \sim 10, \quad \frac{m_t}{m_b} \sim 10^2 \quad (15)$$

whereas the mass splitting between the charged leptons and down quarks remains considerably smaller:

$$\frac{m_e}{m_d} \sim 0.3, \quad \frac{m_\mu}{m_s} \sim 3, \quad \frac{m_\tau}{m_b} \sim 1 \quad (16)$$

so that at large μ the third family is almost unsplit, $m_b \simeq m_\tau$, whereas the first two families are split but $m_d m_s \sim m_e m_\mu$:

Horizontal hierarchy of quark masses exhibites the approximate scaling low

$$m_t : m_c : m_u \sim 1 : \varepsilon_u : \varepsilon_u^2, \quad m_b : m_s : m_d \sim 1 : \varepsilon_d : \varepsilon_d^2 \quad (17)$$

where $\varepsilon_u^{-1} = 200 - 300$ and $\varepsilon_d^{-1} = 20 - 30$. The charged leptons masses have a mixed behaviour:

$$m_\tau : m_\mu : m_e \sim 1 : \varepsilon_e : \varepsilon_e \varepsilon'_e \quad (18)$$

where $\varepsilon_e \sim \varepsilon_d$ and $\varepsilon'_e \sim \varepsilon_u$. One can also exploit experimental information on the quark mixing. The CKM angles exhibit the following hierarvhy:

$$s_{12} \sim \varepsilon_d^{1/2}, \quad s_{23} \sim \varepsilon_d, \quad s_{13} \sim \varepsilon_d^2, \quad (19)$$

which points to the correlations between the quark mass spectrum and mixing pattern. Moreover, there are intriguing relations between masses and mixing angles, such as the well-known formula for the Cabibbo angle $s_{12} = \sqrt{m_d/m_s}$.

It is tempting to think that correlation between the mixing angles and fermion masses is intrinsically connected to the peculiarities of the hypothetical flavour physics at the scale $\mu \sim M_F$, and and the former actually are the functions of the latter. It is also suggestive that the CKM angles have the following "analytic" properties²⁰:

Decoupling. Mixings of the first family with others (s_{12} , s_{13}) vanish in the limit $m_u, m_d \rightarrow 0$. At the next step, when $m_c, m_s \rightarrow 0$, s_{23} also vanishes.

Scaling. All mixing angles s_{12}, s_{13}, s_{23} vanish in the limit when masses of the up and down quarks are proportional to each other: $m_u : m_c : m_t = m_d : m_s : m_b$.

2. Fermion Masses in SUSY GUT: the $SU(5)$ Lessons

2.1. From Love Story to Family Problems

Nowadays the most promising ideas beyond the SM are related to the concepts of supersymmetry (SUSY)²¹ and grand unification theories (GUT)^{22,23,24}. Their relations can be expressed by a simple formula

$$\text{SUSY} + \text{GUT} = \heartsuit \quad (20)$$

Softly broken (at the scale $m_S \sim 1$ TeV) supersymmetry is the only plausible idea that can support the GUT against the gauge hierarchy problem²⁵. The *present* data on $\alpha_3(M_Z)$ and $\sin^2 \theta_W(M_Z)$ are in a remarkable agreement with the *elder* prediction²⁷ of the SUSY $SU(5)$ while exclude the non-supersymmetric $SU(5)$.²⁶ On the other hand, the minimal supersymmetric standard model (MSSM) without GUT is also not in best shape: unification at the string scale gives too small $\sin^2 \theta_W(M_Z)$. In the MSSM the running gauge constants g_3 , g_2 and g_1 given at $\mu = M_Z$ withing their experimental error bars, in their evolution to higher energies join at the scale $M_X \simeq 10^{16}$ GeV.²⁶ Hence, at this scale the $SU(3) \times SU(2) \times U(1)$ symmetry can be consistently embedded into $SU(5)$, which at larger scales can be further extended to larger groups. At this point the elegant GUT and beautiful SUSY, already long time attracted to each other, finally successfully meet.

All these suggest a following paradigm: a basic (string?) “Theory of Everything” below the Planck scale M_P reduces[†] to a SUSY GUT containing the $SU(5)$ subgroup, which then at $M_X \simeq 10^{16}$ GeV breaks down to the $SU(3) \times SU(2) \times U(1)$. Below the scale M_X starts *Great Desert*, with no extra particles besides the known ones and their superpartners. In other words, $\mathcal{X}(M_X) = \mathcal{X}_{\text{MSSM}}$, where $\mathcal{X}_{\text{MSSM}}$ denotes the MSSM particle content containing the chiral superfields of quarks and leptons q_i , l_i , u_i^c , d_i^c , e_i^c ($i = 1, 2, 3$), two Higgs doublets $\phi_{1,2}$, and the gauge superfields of $SU(3) \times SU(2) \times U(1)$. In fact, some extra complete degenerate supermultiplets could populate intermediate scales without spoiling unification of the gauge constants. However, this would affect the RG factors in the Yukawa constant running (see below, eq. (22)).

In the MSSM the fermion masses emerge from the superpotential terms

$$\mathcal{W}_{\text{Yuk}} = \lambda_{ij}^u q_i u_j^c \phi_2 + \lambda_{ij}^d q_i d_j^c \phi_1 + \lambda_{ij}^e l_i e_j^c \phi_1, \quad \mathcal{W}_\nu = \frac{\lambda_{ij}^\nu}{M_P} (l_i \phi_2)(l_j \phi_2) \quad (21)$$

which are straightforward extension of the SM couplings (1) and (2). The Yukawa matrices $\lambda^{u,d,e,\nu}$ remain arbitrary in the MSSM, while presence of two Higgses ϕ_1 and ϕ_2 with VEVs $v_1 = v \cos \beta$ and $v_2 = v \sin \beta$ ($v = 174$ GeV) involves also an additional parameter $\tan \beta = v_2/v_1$.

[†] Without knowing exactly to what is the basic scale of the theory, in the following under the Planck scale we imply a broad range $M_P \sim 10^{17-19}$ GeV, unless it is specified. In particular, this can be the Planck mass $M_{Pl} \simeq 10^{19}$ GeV itself, reduced Planck mass $2 \cdot 10^{18}$ GeV, or a string scale $\sim 3 \cdot 10^{17}$ GeV.

Applied to the flavour problem, grand unification can play an important role in understanding the fermion mass spectrum. It can allow to calculate the Yukawa constants at the scale M_X , or at least somehow constrain them. In order to confront these predictions to the observable mass pattern (7), one has to account for the Yukawa constants RG running down from the scale $M_X \sim 10^{16}$ GeV. By assuming that $\mathcal{X}(M_X) = \mathcal{X}_{\text{MSSM}}$ and considering moderate values of $\tan \beta$, one obtains²⁸:

$$\begin{aligned} m_t &= \lambda_t A_u y^6 v \sin \beta, & m_c &= \lambda_c A_u \eta_c y^3 v \sin \beta, & m_u &= \lambda_u A_u \eta y^3 v \sin \beta \\ m_b &= \lambda_b A_d \eta_b y v \cos \beta, & m_s &= \lambda_s A_d \eta v \cos \beta, & m_d &= \lambda_d A_d \eta v \cos \beta \\ m_\tau &= \lambda_\tau A_e v \cos \beta, & m_\mu &= \lambda_\mu A_e v \cos \beta, & m_e &= \lambda_e A_e v \cos \beta \end{aligned} \quad (22)$$

where the factors A_f account for the gauge boson induced running from the scale M_X down to the SUSY breaking scale $m_S \sim m_t$, y accounts for running induced due to the large top Yukawa constant ($\lambda_t \sim 1$):

$$y = \exp \left[-\frac{1}{16\pi^2} \int_{\ln m_t}^{\ln M_X} \lambda_t^2(\mu) d(\ln \mu) \right] \quad (23)$$

and the factors $\eta_{b,c}$ (or η) encapsulate running from m_t down to $\mu = m_{b,c}$ (or down to $\mu = 1$ GeV for u, d, s). By taking $\alpha_3(M_Z) = 0.11 - 0.13$, we have:

$$\begin{aligned} \eta_b &= 1.5 - 1.6, & \eta_c &= 1.8 - 2.3, & \eta &= 2.1 - 2.8, \\ A_u &= 3.3 - 3.8, & A_d &= 3.2 - 3.7, & A_e &= 1.5 \end{aligned} \quad (24)$$

The RG running for neutrino masses was studied in refs.²⁹.

It is of obvious interest to find a self-consistent, complete and elegant enough example of a SUSY GUT that would provide a *realistic* and *predictive* framework for fermion mass and mixing pattern, and thus could be regarded as a Grand Unification of fermion masses. The naive concept of SUSY GUT solely is not sufficient to achieve this goal, and it should be complemented by other ideas that could further restrict the theory and thus enhance the predictivity.

2.2. Grand DT Hierarchy Problem and small μ Problem

A realistic SUSY GUT should be capable to solve naturally the gauge hierarchy problem. At the level of the SM this is essentially a problem of the Higgs mass stability against radiative corrections (quadratic divergences). It is removed as soon as one appeals to SUSY, which links the scalar masses to those of their fermion superpartners while the latter are protected by the chiral symmetry. In the context of grand unification the gauge hierarchy problem concerns rather the origin of scales: why the weak scale $v \sim M_W$ is so small as compared to the GUT scale M_X , which in itself is not far from the Planck scale M_P . This question is inevitably connected with the doublet-triplet (DT) splitting puzzle²⁵: the Higgs doublets ϕ_1, ϕ_2 embedded in the GUT multiplets are unavoidably accompanied by the colour triplet partners \bar{T}, T . The latter would mediate unacceptably fast proton decay (especially via $d = 5$ operators³⁰) unless their masses are $\sim M_X$.

For example, The Higgs sector of the minimal SUSY $SU(5)$ model consists of chiral superfields in adjoint (24 dimensional) representation Σ and fundamental $(5+\bar{5})$ representations

$$H = (T + \phi_2), \quad \bar{H} = (\bar{T} + \phi_1) \quad (25)$$

with $\phi_{1,2}$ being the MSSM Higgs doublets and \bar{T}, T colour triplets. Superpotential involving these fields has a form:

$$\mathcal{W} = \frac{M}{2}\Sigma^2 + \frac{h}{3}\Sigma^3 + M_H \bar{H}H + f \bar{H}\Sigma H \quad (26)$$

The $SU(5)$ symmetry breaking down to $SU(3) \times SU(2) \times U(1)$ is provided by supersymmetric ground state $\langle \Sigma \rangle = (M/h)\text{diag}(2, 2, 2, -3, -3)$, $\langle \bar{H}, H \rangle = 0$. Then the masses of the T and ϕ superfields are respectively $M_3 = M_H - (3f/h)M$ and $\mu = M_H + (2f/h)M$. So, the light doublet ($\mu \sim M_W$) versus heavy triplet ($M_3 \sim M_X$) requires that $hM_H \approx -2fM$, with the accuracy of about 10^{-14} . Supersymmetry renders this constraint stable against radiative corrections. However, such a “technical solution”²⁵ is nothing but the *Fine Tuning* of parameters in the superpotential.

Natural solution of the DT problem can be provided by the “missing multiplet” mechanism³¹. In this scenario the 24-plet Σ is substituted by the 75-plet Φ of $SU(5)$, and additional heavy superfields $\Psi + \bar{\Psi}$ ($= 50 + \bar{50}$) are introduced. The latter have a bare mass term $M_\Psi \bar{\Psi}\Psi$, $M_\Psi \sim M_X$, while $M_H H \bar{H}$ can be also suppressed by some symmetry. In this case the term $\bar{H}\Phi H$ is absent, but instead the superpotential contains the terms $\bar{H}\Phi\Psi$ and $H\Phi\bar{\Psi}$. Then the triplet components T, \bar{T} in H, \bar{H} can get $\sim M_X$ mass via their mixing to triplet states in $\Psi, \bar{\Psi}$, while the doublets $\phi_{1,2}$, will remain massless since 50-plet does not contain doublet fragment.

Yet another problem is a so-called μ -problem³². The supersymmetric mass term $\mu\phi_1\phi_2$ should be small, and one could think that $\mu = 0$ is a most natural possibility: then the Higgs masses would emerge entirely from the soft SUSY breaking terms with $m_S \sim v$. However, this is excluded experimentally, and μ is required to be $\sim v$ as well. Hence, in SUSY GUTs the gauge hierarchy problem turns into a problem of small (but *not* vanishingly small) μ -term: $\mu \sim m_S$. A realistic SUSY GUT should produce the supersymmetric μ -term in a natural way, without fine tuning of μ to the soft SUSY breaking mass m_S .

2.3. Minimal $SU(5)$ Unification

Already the minimal $SU(5)$ model can provide an important key towards understanding the fermion mass pattern. The quarks and leptons of each family fit into the multiplets

$$\bar{5}_i = (d_i^c + l_i), \quad 10_i = (u_i^c + q_i + e_i^c); \quad i = 1, 2, 3 \quad (27)$$

and the superpotential terms relevant for fermion masses become

$$\mathcal{W}_{\text{Yuk}} = \lambda_{ij}^u 10_i H 10_j + \lambda_{ij}^d 10_i \bar{H} \bar{5}_j, \quad \mathcal{W}_\nu = \frac{\lambda_{ij}^\nu}{M_P} (\bar{5}_i H) (H \bar{5}_j) \quad (28)$$

At the GUT scale M_X these terms reduce to the MSSM couplings (21) with $\hat{\lambda}_{ij}^e = \hat{\lambda}_{ji}^d$, and hence $\lambda_{d,s,b} = \lambda_{e,\mu,\tau}$.

The $\lambda_b = \lambda_\tau$ unification³³ is definitely a *Grand Prix*. After accounting for the RG running (22), it translates into $m_b/m_\tau = y\eta_b A_d/A_e \sim 3y$. When I get to the bottom I go back to the top³⁴: for $\lambda_t \ll 1$ ($y = 1$) this explains basic factor of 3 difference between the masses of bottom and tau. However, the more precise comparison of m_b and m_τ within the uncertainties in (7) points that $y < 1$. This in turn implies a rather large λ_t (≥ 1), in which case the top mass is fixed by its infrared limit³⁵

$$M_t = (190 - 210) \sin \beta \text{ GeV} = 140 - 210 \text{ GeV} \quad (29)$$

Thus, the minimal SUSY $SU(5)$ model explains the principal origin of the bottom quark mass and nicely links it to the large value of the top mass, within the range indicated by the present data⁷.

Unfortunately, the other predictions $\lambda_s = \lambda_\mu$ and $\lambda_d = \lambda_e$ are wrong: they imply $m_s/m_d = m_\mu/m_e \simeq 200$ – *c'est la vie!* In addition, there is no explanation neither for the fermion mass hierarchy nor for the CKM mixing pattern: the Yukawa matrices λ^u and λ^d remain arbitrary and there is no reason for their alignment. Therefore, one is forced to go beyond the minimal $SU(5)$ model and implement new ideas that could shed some more light on the origin of fermion masses and mixing.

2.4. Tools for Fermion Mass Models: Oldies but Goldies

Below we briefly review some ideas that can be regarded as a *Modus Operandi* for the predictive model building.

- *Mass matrix textures*. Relations between the fermion masses and CKM angles can be obtained by considering the mass matrix ansatzes with reduced number of free parameters. In particular, certain elements in the Yukawa constant matrices can be put to zero (so called “zero textures”)^{36,37}. One of the most popular ansatzes was suggested by Fritzsch³⁷:

$$\hat{\lambda}_f = \begin{pmatrix} 0 & A_f & 0 \\ A'_f & 0 & B_f \\ 0 & B'_f & C_f \end{pmatrix}, \quad f = u, d, e \quad (30)$$

It implies that the fermion mass generation starts from the 3^{rd} family (C is assumed to be a largest entry in eq. (30)) and proceeds to lighter families through the mixing terms. One can assume further that

$$|A'_f| = |A_f|, \quad |B'_f| = |B_f|. \quad (31)$$

Then, if neglect the phase factors, the total number of parameters for each matrix $\hat{\lambda}_{u,d,e}$ is reduced to 3, i.e. just the number of the quark and lepton species. This allows to express the quark mixing angles in terms of their mass ratios:

$$s_{12} = \left| \sqrt{\frac{\lambda_d}{\lambda_s}} - e^{i\delta} \sqrt{\frac{\lambda_u}{\lambda_c}} \right|, \quad s_{23} = \left| \sqrt{\frac{\lambda_s}{\lambda_b}} - e^{i\kappa} \sqrt{\frac{\lambda_c}{\lambda_t}} \right|, \quad s_{13} = \sqrt{\frac{\lambda_u}{\lambda_c}} s_{23} \quad (32)$$

where δ is a CP-violating phase and κ is some unknown phase. In particular, when $\delta \sim 1$, we obtain $s_{12} \approx \sqrt{m_d/m_s}$. Unfortunately, the value of s_{23} in eq. (32) is not compatible with the large top mass, neither in ordinary nor in supersymmetric cases³⁸. There are however some simple modifications³⁹ of the ansatz which could still agree to the experimental data. For example, if $|B'_d| = 2|B_d|$, one obtains a consistent value for s_{23} (see below, eq. (45)). A dedicated analysis of possible zero-textures can be found in ref.⁴⁰.

- *Radiative mechanism.* The observed mass hierarchy of about 1 – 2 orders of magnitude between neighbouring families makes attractive the idea that radiative corrections may be responsible for mass generation. Masses of the light fermions could arise as a radiative effect from the tree-level masses of heavy family^{41,42}. Namely, if due to some reasons only the 3rd family fermions have tree-level masses, the 2nd family masses emerge at the 1-loop level and the 1st family becomes massive only at 2-loops, then the inter-family hierarchy will have a shape of eqs. (17),(18), with $\varepsilon_f \sim (g_f^2/16\pi^2)$ and g_f being typical coupling constants of the order of 1.

Radiative models^{41,42} provide a rather qualitative explanation to the fermion mass hierarchy, and generally fail in predictivity. In particular, they cannot reproduce zero textures for mass matrices. Moreover, it is very difficult to obtain a quantitatively correct picture (e.g. $s_{12} \simeq \sqrt{m_d/m_s}$ contradicts to perturbativity), and also to avoid dangerous flavour changing phenomena⁴³.

A consistent and predictive radiative approach was suggested in⁴⁴, where fermion mass hierarchy is first radiatively generated in the ‘hidden’ sector of the heavy vectorlike fermions and the transferred in an inverted way to the usual quarks and leptons by means of the “universal seesaw” mechanism. However, these models cannot be valid in the SUSY GUT context: at scales much larger than m_S the loop corrections will be suppressed by supersymmetry.

Nevertheless, this mechanism leaves the following important message: the lighter fermion masses could be due to the higher order operators. In gauge theories the vanishing of certain mass terms at the tree-level can occur as a consequence of the representation content of the fields in the theory, or due to some inter-family symmetry. In radiative scenario these mass terms then can emerge in the effective action as operators of dimension $d > 4$ (i.e. involving more than one scalar leg).

Within SUSY frames one could think of some tree level mechanism that could generate the relevant effective operators with successively increasing dimension, and thus explain the observed mass hierarchy.

- *Higher order (non-renormalizable) operators (HOP).* Indeed, there is no physical reason for suppressing the higher order non-renormalizable terms scaled by inverse power of the Planck scale M_P . In fact, we have already introduced one such a term in (28) for the neutrino mass generation. In the minimal $SU(5)$ theory one can consider⁴⁵ the higher dimension terms involving the 24-plet Σ :

$$\frac{1}{M_P} 10 \Sigma H 10 + \frac{1}{M_P} 10 \Sigma \bar{H} \bar{5}, \quad \frac{1}{M_P^2} 10 \Sigma^2 H 10 + \frac{1}{M_P^2} 10 \Sigma^2 \bar{H} \bar{5}, \quad \dots \quad (33)$$

which can be relevant for the light fermion masses. Below the scale $\langle \Sigma \rangle \sim M_X$ they contribute to the Yukawa couplings (21) with the magnitudes $\sim \varepsilon_X, \varepsilon_X^2$ etc., $\varepsilon_X = M_X/M_P$. This suggests that maybe the renormalizable couplings (28) fix only the third family masses, thus maintaining the $\lambda_b = \lambda_\tau$ unification, and masses of the lighter families emerge entirely from the HOPs like (33). In this case one can avoid the *wrong* predictions $\lambda_{d,s} = \lambda_{e,\mu}$ of the minimal $SU(5)$, since the tensor product 24×5 contains also the effective 45-plet.

Addressing the idea of mass matrix textures and also modifying the Higgs content of the theory, one can build more realistic and predictive models. For example, consider a “missing partner” model with 75-plet Φ instead of the 24-plet Σ , including also a singlet Y with VEV V_Y . Let us also assume that due to some (inter-family) symmetry G_H only the third family masses emerge from the direct Yukawa terms, while other entries are induced through the HOPs, and the lowest order terms in the superpotential allowed by G_H are the following:

$$\begin{aligned}\mathcal{W}_{\text{up}} &= 10_3 H 10_3 + \frac{1}{M_P} 10_2 (\Phi H)_{45} 10_3 + \frac{Y}{M_P^2} 10_1 (\Phi H)_{45} 10_2 \\ \mathcal{W}_{\text{down}} &= 10_3 \bar{H} \bar{5}_3 + \frac{1}{M_P} 10_2 (\Phi \bar{H})_{\bar{45}} \bar{5}_2 + \frac{Y}{M_P^2} (10_1 \bar{H} \bar{5}_2 - 10_2 \bar{H} \bar{5}_1)\end{aligned}\quad (34)$$

(the order 1 coupling constants in each term are omitted). Then the Yukawa constant matrices at GUT scale obtain the following pattern:

$$\hat{\lambda}^u = \begin{pmatrix} 0 & -C & 0 \\ C & 0 & B \\ 0 & -B & A \end{pmatrix} \quad \hat{\lambda}^d = \begin{pmatrix} 0 & -F e^{i\delta} & 0 \\ F e^{-i\delta} & E & 0 \\ 0 & 0 & D \end{pmatrix} \quad \hat{\lambda}^e = \begin{pmatrix} 0 & -F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix} \quad (35)$$

It is a key moment that the tensor product 75×5 can induce the fermion masses only via the 45 channel. This creates the relative Clebsch coefficient -3 between the (2,2) entries in $\hat{\lambda}^d$ and $\hat{\lambda}^e$. On the other hand, 10×10 contains 45 only in antisymmetric tensor product, so that no diagonal entries can be induced in $\hat{\lambda}^u$ while the (1,2) and (2,3) entries are antisymmetric. Interestingly, this also implies that quark couplings to the triplet component T in $H, q_i q_j T$, being symmetric cannot emerge from these antisymmetric couplings. This will lead to partial suppression of the dangerous $d = 5$ operators for the proton decay (see below, in section 5).

Except the (1,2) entry in $\hat{\lambda}^d$, all matrix elements in (35) can be made real by proper phase redefinition of quark and lepton fields. Hence one is left with 8 parameters (A, B, C, D, E, F, δ and $\tan \beta$) versus 14 observables (9 fermion masses, 3 mixing angles, CP-phase and $\tan \beta$), and thus 6 predictions can be obtained.

The ansatz (35) was initially suggested by Georgi and Jarlskog⁴⁶ in ordinary $SU(5)$ with the tree level Yukawa couplings involving the Higgs 5- and 45-plets. Its predictions in the supersymmetric case were elaborated at length in ref.⁴⁷. From (35) one obtains (at the scale $\mu \sim M_X$) the Yukawa constant relations

$$\lambda_b = \lambda_\tau, \quad 3(\lambda_s - \lambda_d) = \lambda_\mu - \lambda_e, \quad \lambda_d \lambda_s = \lambda_e \lambda_\mu \quad (36)$$

and the following expressions for the CKM mixing angles

$$s_{12} = \left| \sqrt{\frac{\lambda_d}{\lambda_s}} - e^{i\delta} \sqrt{\frac{\lambda_u}{\lambda_c}} \right|, \quad s_{23} = \sqrt{\frac{\lambda_c}{\lambda_t}}, \quad s_{13} = \sqrt{\frac{\lambda_u}{\lambda_c}} s_{23} \quad (37)$$

After accounting for the RG running, these predictions can be tested for the low energy (experimental) observables. We know that the Yukawa unification $\lambda_b = \lambda_\tau$ motivates the large value of λ_t . This in turn can render the value of s_{23} compatible with experimental data, provided that $\sin \beta \sim 1$.⁴⁷ For the light quark masses one obtains $m_s \sim 150$ MeV and $m_d/m_s = 9m_e/m_\mu \approx 1/22$, in agreement with the current estimates (8). In addition, for $\delta \sim 1$ we have $s_{12} \approx \sqrt{m_d/m_s} \approx 0.2$.

Besides reproducing the predictive power of the ansatz^{46,47}, the operator structure in (34) can explain the origin of the fermion mass hierarchy. Indeed, we have:

$$\begin{aligned} \lambda_t : \sqrt{\lambda_c \lambda_t} : \sqrt{\lambda_u \lambda_c} &= A : B : C \sim 1 : \varepsilon_X : \varepsilon_X^2 \varepsilon_Y \\ \lambda_\tau : \lambda_\mu : \sqrt{\lambda_e \lambda_\mu} &= D : E : F \sim 1 : \varepsilon_X : \varepsilon_X \varepsilon_Y \end{aligned} \quad (38)$$

where $\varepsilon_X = M_X/M_P$ and $\varepsilon_Y = V_Y/M_X$. The fermion mass pattern can be reproduced if $\varepsilon_{X,Y} \sim 1/15$ or so; we have $\lambda_t/\lambda_c \sim \lambda_c/\lambda_u \sim \lambda_\mu/\lambda_e \sim 200$, as in eqs. (17) and (18). Then from $M_X \simeq 2 \cdot 10^{16}$ GeV we obtain $M_P \sim 3 \cdot 10^{17}$ GeV (string scale?), and $V_Y \sim 10^{15}$ GeV.

- *Heavy fermion exchanges* (HFE). Higher order operators can be induced through the renormalizable interactions, as a result of integrating out the hypothetical super-heavy particles^{48,49,50,51}. In other words, the quark and lepton masses can be induced through their mixings with the superheavy fermions, in a direct analogy to the celebrated seesaw mechanism for neutrinos³. Let us recall that in this scenario the RH neutrino ν^c is essentially a heavy neutral fermion with a Majorana mass $M \gg v$. The mass of physical (LH) neutrino emerges via its mixing term (=Dirac mass term) to the RH neutrino. Namely, at scales below M a diagram mediated by exchange of ν^c reduces to the effective operator (2).

One can introduce also a vector-like set of *charged* heavy fermions, having the same quantum numbers as the usual quark and lepton species. In the following we refer them to as F -fermions. Their exchanges can induce effective HOPs cutoff by a scale $M \sim M_F$ (not necessarily M_P). For example, the operators like (33) or (34) can be induced by F -fermions in representations^{49,50}

$$X_k + \bar{X}_k, \quad \bar{V}_l + V_l, \quad \text{etc.} \quad (39)$$

(following ref.⁵⁰, we use roman numerals $V = 5$ and $X = 10$ to denote their dimensions). Such fermions with mass $\sim M_P$ can exist in string theories. Typically they emerge also in the context GUT theories larger than $SU(5)$. E.g., 27-plet of the E_6 theory, in addition to $\bar{5} + 10$ contains also extra $\bar{V} + V$ states. In $SU(11)$ model⁵², after the $SU(11)$ symmetry breaking down to $SU(5)$, a number of F -states (39) emerge

along with three chiral families of $\bar{5} + 10$. In spirit of the *survival hypothesis*⁵², they should get masses at the scales of the larger GUT symmetry breaking to $SU(5)$. Hence, generically the F -fermion masses and correspondingly cutoff scales of the effective HOPs, can vary from M_P to M_X .

The HOPs induced by the HFE mechanism can be more instructive for the fermion mass model building than the *ad hoc* introduced non-renormalizable operators. The HFE can produce the relevant operators in a rather selective way, and can fix the Clebsch factors between the Yukawa constants in dependence on the F -particles representations. Examples demonstrating advantages of the HFE mechanism will be given in next sections.

In the literature the HFE mechanism, in its simplest form^{49,53} which is reminiscent of the neutrino seesaw scheme, is also known as “universal seesaw”. For its applications, see e.g. ref.⁵⁴ and references therein.

- *Horizontal (inter-family) symmetries.* The mass matrix textures can arise from the spontaneously broken horizontal symmetry between the fermion families. Consider, for example, model with all quark and lepton states transforming as triplets $f_\alpha = (q, l, u^c, d^c, e^c)_\alpha$ of the horizontal $SU(3)_H$ symmetry⁵⁵, ($\alpha = 1, 2, 3$ is a family index). In order to avoid proliferation of Higgs doublets in non-singlet horizontal representations, the Higgs ϕ should be a singlet of the $SU(3)_H$. Extra light (with masses $\sim v$) Higgs doublets would spoil the natural suppression of the FCNC¹⁵ and would also destroy the gauge coupling unification, thus preventing any attempt to embed the model in SUSY GUT.

Such a horizontal symmetry does not allow quarks and leptons to have renormalizable Yukawa couplings. Hence, the fermion mass generation is possible only after the $SU(3)_H$ breaking, through the HOPs involving some “horizontal” Higgses inducing this breaking at scales $V_H \gg v$. This suggests that observed mass hierarchy may emerge due to the hierarchy in the $SU(3)_H$ symmetry breaking.

One can introduce horizontal scalars $\chi^{\alpha\beta}$ in the two-index symmetric or antisymmetric representations: say a sextet $\chi_3^{\{\alpha\beta\}}$ and two triplets $\chi_{1,2}^{[\alpha\beta]} \sim \varepsilon^{\alpha\beta\gamma} \chi_\gamma$. Their VEV pattern can be chosen so that the sextet χ_3 has a VEV V_{33} towards (3,3) component, and triplets χ_2 and χ_1 have the smaller VEVs V_{23} and V_{12} directed towards 1st and 3rd components. Thus, the total matrix of horizontal VEVs has a form⁵⁰:

$$\hat{V}_H = \sum_k \langle \chi_k \rangle = \begin{pmatrix} 0 & V_{12} & 0 \\ -V_{12} & 0 & V_{23} \\ 0 & -V_{23} & V_{33} \end{pmatrix}, \quad V_{33} \gg V_{23} \gg V_{12} \quad (40)$$

Then fermion masses can be induced by HOPs involving the horizontal Higgses χ_k . For example, in the context of the $SU(5) \times SU(3)_H$ theory with fermions in representations $(\bar{5} + 10)_\alpha$. The relevant operators at lowest order are the following⁵⁰:

$$\mathcal{W}_{\text{up}} = \frac{g_k^u \chi_k^{\alpha\beta}}{M} 10_\alpha H 10_\beta, \quad \mathcal{W}_{\text{down}} = \frac{g_k^d \chi_k^{\alpha\beta}}{M} 10_\alpha \bar{H} \bar{5}_\beta, \quad \mathcal{W}_\nu = \frac{g_k^\nu \chi_k^{\alpha\beta}}{M^2} (\bar{5}_\alpha H) (H \bar{5}_\beta) \quad (41)$$

These operators (41) can be induced through the HFE mechanism using the F -fermions $X^\alpha + \bar{X}_\alpha$ and $\bar{V}^\alpha + V_\alpha$, respectively in $(10 + \bar{5}, \bar{3})$ and $(\bar{10} + 5, 3)$ representations of $SU(5) \times SU(3)_H$. Certainly, in this case the VEV pattern (40) is directly reflected in the light fermion Yukawa matrices, and the fermion mass hierarchy follows to the $SU(3)_H$ symmetry breaking hierarchy. We refer this case as to a *direct hierarchy* pattern. In particular, the VEV pattern (40) leads directly to the Fritzsch texture (30) considered above.[‡]

However, the HFE mechanism can suggest also another possibility, known as the *inverse hierarchy* pattern⁴⁹. Namely, the F -fermions can be introduced as $(X + \bar{X})^\alpha$, etc. so that their invariant mass terms are forbidden by the $SU(3)_H$ symmetry. Then they can get masses after the horizontal symmetry breaking, via the Yukawa couplings $X\chi\bar{X}$ etc. On the other hand, these fermions can mix the $(\bar{5} + 10)_\alpha$ states through the $SU(3)_H$ invariant couplings like $10A\bar{X}$, $10HX$, $\bar{5}\bar{H}X$ etc., where A is some dimensional parameter (or alternatively the Higgs 24 or 75 of $SU(5)$). Then, for $A \leq V_H$, the decoupling of the heavy states leads to effective operators⁴⁹

$$\mathcal{W}_{\text{Yuk}} = 10_\alpha \frac{AH}{\langle \sum g_k^u \chi_{\alpha\beta}^k \rangle} 10_\beta + 10_\alpha \frac{A\bar{H}}{\langle \sum g_k^d \chi_{\alpha\beta}^k \rangle} \bar{5}_\beta \quad (42)$$

which *project* the VEV pattern (40) on the fermion mass structure in the inverted way. The possible implications of the inverse hierarchy horizontal $SU(3)_H$ models were discussed in refs.^{56,57}.

The $SU(3)_H$ symmetry is attractive since it unifies all families. Within the same lines one can consider the models with a reduced horizontal symmetry $SU(2)_H$ acting between first two families; then third family can get mass from the direct Yukawa couplings. Several other possibilities can be also envisaged, including discrete or abelian horizontal symmetries⁵⁸.

- *P, CP, PQ and other flavour-blind symmetries.* Flavour-blind symmetries like spontaneously broken P and CP parities can also help in constraining the mass matrices. In particular, in the context of the $L - R$ symmetric model mass matrices should be Hermitian due to P-parity. For the Fritzsch ansatz (30) this would imply a condition (31). The spontaneously broken CP-invariance could constrain the complex phases of the Yukawa constants and thus enhance predictivity⁶¹.

P or CP can provide a solution to the strong CP-problem without introducing an axion, *a la* Nelson-Barr mechanism⁵⁹. Such models based on “universal seesaw” were suggested in⁶⁰, where the Θ -term automatically vanishes at the tree level and

[‡] The $b - \tau$ Yukawa unification requires that χ_3 is the $SU(5)$ singlet. As for triplets $\chi_{1,2}$, they should have a non-trivial $SU(5)$ assignment, say 24 or 75. Only in this case they can couple 10's in \mathcal{W}_{up} and thus generate antisymmetric off-diagonal entries in $\hat{\lambda}_u$ (antisymmetric tensor product 10×10 contains only the 45-channel, which then can be confronted by the effective 45 in 24×5 product of the scalar fields). On the other hand, the effective $\bar{45}$ in $\mathcal{W}_{\text{down}}$ will allow to split the down quark and charged lepton Yukawa constants in first two families. Alternatively, for purposes of economy, one could assume that $\chi_{1,2}$ are $SU(5)$ singlets, but by some symmetry reasoning they appear only in the next order terms together with Σ or Φ , like $(1/M^2)10\chi_{1,2}\Sigma H10$, etc.

emerges to be naturally small in the loop corrections. Alternatively, for the solution of the strong CP-problem one can introduce the Peccei-Quinn (PQ)⁶² type symmetries, which in addition could further restrict the mass matrix structure. In particular, in the horizontal $SU(3)_H$ symmetry models the PQ symmetry can be naturally related to the phase transformation of the horizontal scalars χ .^{49,50,56} In this case axion appears to be simultaneously a majoron and familon.

We conclude this section by demonstrating a $SU(5) \times SU(3)_H$ model which allows to properly correct the Fritzsch texture maintaining its predictive power. Let us introduce, along with the already familiar horizontal sextet χ_3 and triplets $\chi_{1,2}$, also the $SU(3)_H$ octet scalar Λ_β^α with the VEV $\sim \text{diag}(1, 1, -2)$ towards the λ_8 direction. Let us also assume a ‘flavour-blind’ discrete symmetry Z_2 , under which the Λ and $\bar{5}_\alpha$ states change the sign while all other states are invariant. Then operator \mathcal{W}_{up} in (41) is still effective for the up quark masses, but $\mathcal{W}_{\text{down}}$ is forbidden now by the Z_2 symmetry. However, it can be replaced by the next order operator:

$$\mathcal{W}_{\text{down}} = \frac{g_k^d}{M^2} 10_\alpha (\Lambda \chi_k)^{\alpha\beta} \bar{H} \bar{5}_\beta \quad (43)$$

which can be obtained through the ‘double’ exchange mediated by the F -fermions $\bar{V}^\alpha + V_\alpha$ (Z_2 singlets) and $\bar{V}'^\alpha + V'_\alpha$ (which also change the sign under Z_2). Then, by taking into account the VEV pattern (40), we arrive to the following textures:

$$\hat{\lambda}^u = \begin{pmatrix} 0 & A_u & 0 \\ -A_u & 0 & B_u \\ 0 & -B_u & C_u \end{pmatrix} \quad \hat{\lambda}^d = \begin{pmatrix} 0 & A_d & 0 \\ -A_d & 0 & B_d \\ 0 & 2B_d & C_d \end{pmatrix} \quad \hat{\lambda}^e = \begin{pmatrix} 0 & -A_e & 0 \\ A_e & 0 & 2B_e \\ 0 & B_e & C_d \end{pmatrix} \quad (44)$$

Thus, the condition $|B_d| = |B'_d|$ is avoided and instead of eq. (32) we obtain

$$s_{12} = \left| \sqrt{\frac{\lambda_d}{\lambda_s}} - e^{i\delta} \sqrt{\frac{\lambda_u}{\lambda_c}} \right|, \quad s_{23} = \left| \sqrt{\frac{\lambda_s}{2\lambda_b}} - e^{i\kappa} \sqrt{\frac{\lambda_c}{\lambda_t}} \right|, \quad s_{13} = \sqrt{\frac{\lambda_u}{\lambda_c}} s_{23} \quad (45)$$

(notice the factor 2 in expression for s_{23}). Thus all predictions but for s_{23} are the same as in the Fritzsch ansatz, and now the prediction for s_{23} can perfectly fit its experimental value (9).

In Section 4 we will demonstrate that flavour-blind symmetries can do much better job without any horizontal symmetry.

3. Fermion masses in SUSY $SO(10)$

3.1. $SO(10)$ Unification

$SO(10)$ is a smallest group in which all the fermions in one family fit into one irreducible representation 16. In addition to the quark and lepton states of the SM, it includes also RH neutrino ν^c (singlet of $SU(5)$). Thus, $SO(10)$ can in principle relate all Yukawa matrices $\hat{\lambda}^{u,d,e,\nu}$ by the $SO(10)$ Clebsch factors and thus reduce the number of fundamental parameters in the fermion sector.

The $SO(10)$ symmetry can break down to $SU(3) \times SU(2) \times U(1)$ via two interesting channels: $SO(10) \rightarrow SU(5)$ or $SO(10) \rightarrow G_{422} = SU(4) \times SU(2) \times SU(2)'$. For the symmetry breaking purposes one has to introduce a set of Higgses in representations 54, 45 and $16 + \bar{16}$, which we recall later on as S , A and $\psi + \bar{\psi}$ fields. Their contents in terms of the $SU(5)$ and G_{422} subgroups are the following:

$$\begin{aligned} SU(5) : \quad & 54 = 24 + 15 + \bar{15}, \quad 45 = 1 + 24 + 10 + \bar{10}, \quad 16 = 1 + \bar{5} + 10 \\ G_{422} : \quad & 54 = (1, 1, 1) + (3, 3, 1) + (1, 1, 20') + (2, 2, 6), \quad 16 = (4, 2, 1) + (\bar{4}, 1, 2), \\ & 45 = (3, 1, 1) + (1, 3, 1) + (1, 1, 15) + (2, 2, 6) \end{aligned} \quad (46)$$

The VEVs of $\psi, \bar{\psi}$ towards the $SU(5)$ singlet component reduces $SO(10)$ to $SU(5)$, while the VEV of S contains only the G_{422} singlet. The $SO(10)$ symmetry does not allow S and $\psi, \bar{\psi}$ superfields to have renormalizable couplings to each other in superpotential. However, A fields can couple both S and ψ . One can introduce the Higgs 45-plets of the following types: “simple” fields A_{BL} having the VEV V_{BL} only towards the $(15, 1, 1)$ fragment, A_R with VEV V_R on the $(1, 1, 3)$ fragment, A_X with VEV V_X towards the $SU(5)$ singlet component and A_Y with VEV V_Y orthogonal to V_X , and a “general” one \tilde{A} having a VEV \tilde{V} shared by both $(15, 1, 1)$ and $(1, 1, 3)$ components. The VEV orientation of the 45-plets are determined by their couplings to the S and ψ type superfields (see e.g. ref.⁶⁴). In particular, \tilde{A} has VEVs towards both $(15, 1, 1)$ and $(1, 1, 3)$ fragments if superpotential includes the both terms $\tilde{A}^2 S$ and $\psi \tilde{A} \bar{\psi}$. As for A_{BL} and A_R , in order to ensure the strict ‘zeroes’ in their VEVs, they should couple only to S but not to $\psi \bar{\psi}$. On the contrary, A_X should have no coupling with S but only $\psi A_X \bar{\psi}$. The trilinear terms like $A_{BL} A_R A_X$ are also necessary in order to evade the unwanted Goldstone modes.

As for the Higgs doublets $\phi_{1,2}$, they fit into the 10-plet H of $SO(10)$:

$$\begin{aligned} SU(5) : \quad & 10 = \bar{5}(\bar{T}, \phi_1) + 5(T, \phi_2), \\ G_{422} : \quad & 10 = \phi(1, 2, 2) + \mathcal{T}(6, 1, 1) \end{aligned} \quad (47)$$

while three fermion families are arranged in chiral superfields 16_i ($i = 1, 2, 3$):

$$\begin{aligned} SU(5) : \quad & 16_i = \bar{5}(d^c, l)_i + 10(u^c, q, e^c)_i + 1(\nu^c)_i, \\ G_{422} : \quad & 16_i = f_i(4, 2, 1) + f_i^c(\bar{4}, 1, 2) \end{aligned} \quad (48)$$

For the fermion mass generation via the HFE mechanism one can also introduce superheavy families $16_k^F + \bar{16}_k^F$ ($k = 1, 2, \dots$):

$$\begin{aligned} SU(5) : \quad & 16_k^F = (X + \bar{V} + I)_k, \quad \bar{16}_k^F = (\bar{X} + V + \bar{I})_k \\ G_{422} : \quad & 16_k^F = \mathcal{F}_k(4, 2, 1) + F_k^c(\bar{4}, 1, 2), \quad \bar{16}_k^F = \mathcal{F}_k^c(\bar{4}, 2, 1) + F_k(4, 1, 2) \end{aligned} \quad (49)$$

In order to maintain the gauge coupling unification, we assume that all VEVs V_S, V_ψ, V_{BL} , etc. are of the order of $M_X \simeq 10^{16}$ GeV, and below this scale SUSY $SO(10)$ theory reduces to the MSSM with three fermion families f_i and the MSSM

Higgses $\phi_{1,2}$ (i.e. $\mathcal{X}(M_X) = \mathcal{X}_{\text{MSSM}}$). The field A_{BL} can serve for the solution of the DT splitting problem through the “missing VEV” mechanism^{63,64}. In this way the Higgs doublets $\phi_{1,2}$ can be kept light while their colour triplet partners \bar{T}, T acquire the $O(M_X)$ mass.

3.2. Direct Hierarchy Approach: $t - b - \tau$ Unification and Large $\tan \beta$

One can assume that the masses of the heavy family emerge from a single renormalizable operator with the $O(1)$ coupling constant:

$$O_{33} = 16_3 H 16_3 \quad (50)$$

This leads to the unification of the Yukawa couplings $\lambda_t = \lambda_b = \lambda_\tau$, which is reminiscent of the unification of the three gauge couplings g_1, g_2 and g_3 .⁶⁵

The other fermion masses emerge from the higher order operators. In order to achieve predictivity, one has to appeal to the idea of zero textures. Recently various mass matrix ansatzes have been considered in the SUSY $SO(10)$ framework and several interesting (and testable) predictions were obtained^{66,67}. In particular, in ref.⁶⁶ the so called “22” textures were studied:

$$\hat{\lambda}^f = \begin{pmatrix} 0 & z'_f C & 0 \\ z_f C & y_f E e^{i\omega} & x'_f B \\ 0 & x_f B & A \end{pmatrix}, \quad f = u, d, e, (\nu) \quad (51)$$

where z_f, z'_f etc. are the Clebsch factors distinguishing different fermions. Once the (3,3) entry emerges from (50), its Clebsch is independent of $f = u, d, e$. Other Clebsches can be fixed by the structures of the higher order operators O_{23}, O_{22} etc. generating corresponding entries. In order to unambiguously fix these Clebsches, one has to accept the following rules of the game: (i) the relevant operators are obtained by means the HFE mechanism by exchange of heavy $16^F + \overline{16}^F$ states (49). These either have an invariant mass M or acquire mass through the Yukawa couplings to the Higgs 45-plets; (ii) all 45-plets involved into the game have ‘simple’ VEV structure (i.e. there are only 45-plets $A_{BL,R,X,Y}$ and no ‘general’ one \tilde{A}). For example, in one of the models of ref.⁶⁶ these operators have the following form, obtained by exchanges of 9 heavy species ($16^F + \overline{16}^F$):

$$O_{23} = 16_2 \frac{A_Y}{A_X} H \frac{A_Y}{A_X} 16_3, \quad O_{22} = 16_2 \frac{A_X}{M} H \frac{A_{BL}}{A_X} 16_2, \quad O_{12} = 16_1 \left(\frac{A_X}{M} \right)^3 H \left(\frac{A_X}{M} \right)^3 16_2 \quad (52)$$

Then the fermion mass and mixing pattern is determined by 6 parameters A, B, C, E, ω and $\tan \beta$ which describe 14 observables. Thus 8 predictions can be obtained. A typical strategy for analysing these predictions is to fix six better known quantities: masses of the charged leptons (m_e, m_μ, m_τ), c, b quark masses (m_c, m_b) and the Cabibbo angle s_{12} as input parameters, and deduce precise predictions for other, less known quantities: $m_{u,d,s}, M_t, s_{12}, s_{13}$ and CP-phase δ . In ref.⁶⁶ a plethora of all possible operator structures was scanned, and it was shown that only very few of them

could fit the observed pattern of the fermion masses and mixings. It is noteworthy that in all consistent cases the Clebsches y_i satisfy $y_u : y_d : y_e = 0 : 1 : 3$, the form familiar from the Georgi-Jarlskog ansatz⁴⁶. For the list of the testable predictions of the ansatzes (51) one can address ref.⁶⁶.

Needless to say, that one needs to invoke a very complex set of inter-family as well as flavour-blind symmetries which from one hand would enforce zeros in (51) and, on the other hand, would fix the needed pattern of ‘non-zero’ operators. These symmetries should also fix the Higgs superpotential terms in such a way that would force the Higgs 45-plets to have the VEVs of the desired ‘simple’ pattern: only X , Y , BL and R directions, and also would implement the “missing VEV” mechanism⁶³ for the DT splitting.

A key message of the *direct hierarchy* models^{65,66} is that third family plays a role of the Yukawa unification point at the GUT scale. It gets mass from the tree-level Yukawa term which fixes $\lambda_t = \lambda_b = \lambda_\tau$ at the GUT scale. Then mass generation proceeds to the lighter families through the smaller terms induced by the HOPs. The large splitting of the top and bottom masses can be reconciled only at the price of extremely large $\tan\beta$, of about two orders of magnitude. This can be achieved by certain tuning of parameters in the Higgs sector⁶⁸. However, it becomes then rather surprising that despite the giant $\tan\beta$, m_c/m_s is about 10 times less than m_t/m_b while the first family is almost unsplit, $m_u \sim m_d$. Such a situation is reminiscent of a fine tuning, and it deserves a judicious selection of the $SO(10)$ Clebsch coefficients. In particular, the operator O_{12} in (52), which implies $z_u : z_d : z_e = -\frac{1}{27} : 1 : 1$ ($z'_f = z_f$), appears to be the unique consistent one⁶⁶.

3.3. Inverse Hierarchy Approach: $u - d - e$ Unification and Small $\tan\beta$

The fermion mass pattern (15), (16) suggests that the first family might play a key role in understanding the structure of flavour: a role of the mass unification point. Indeed, running masses of the first family exhibit an approximate $SO(10)$ symmetry: $m_e \sim m_u \sim m_d$ with splitting of about a factor of 2, while the heavier families strongly violate it. In the context of small $\tan\beta$ this may indicate that the $SO(10)$ Yukawa unification holds for the constants $\lambda_{u,d,e}$ rather than for $\lambda_{t,b,\tau}$.

Such a possibility can be indeed realized by HFE mechanism in the *inverse hierarchy* approach⁴⁹. The inter-family hierarchy could first emerge in a sector of F -fermions (49) and then be transferred inversely to ordinary quarks and leptons by means of the ‘universal seesaw’. With this picture in mind, it is suggestive to think that the 1st family masses are unsplit since they are related to an energy scale $M_1 > M_X$ at which the $SO(10)$ symmetry is still good, while the masses of the 2nd and 3rd family are respectively related to lower scales $M_{2,3} \leq M_X$, at which $SO(10)$ is no longer as good.

A predictive inverse hierarchy framework in SUSY $SO(10)$ was suggested in ref.⁶⁹. This model involves the ‘simple’ 45-plets A_{BL} with VEV V_{BL} on the $(15, 1, 1)$ fragment, A_R having a VEV V_R towards the $(1, 1, 3)$ component, and a ‘general’ one \tilde{A} with the VEV \tilde{V} shared by both $(15, 1, 1)$ and $(1, 1, 3)$ directions in terms of (46). All these VEVs are assumed to be of the order of $M_X \sim 10^{16}$ GeV, and the only

scale beyond is a string one $M_P = 10^{17.5}$ GeV. The field A_{BL} is used entirely for the DT splitting via the “missing VEV” mechanism, whereas A_R and \tilde{A} participate the fermion mass generation.

It was assumed that 16_i (48) have no direct Yukawa couplings $16_i H 16_j$ by certain (inter-family) symmetry reasons, and their masses *all* emerge through the ‘seesaw’ mixing with three heavy families $(16^F + \overline{16}^F)_i$ (49). The relevant terms in superpotential are chosen as

$$\mathcal{W}_{\text{mix}} = \Gamma_{ij} 16_i H 16_j^F + \frac{G_{ij}}{M_P} (\overline{16}_i^F A_R) (A_R 16_j) \quad (53)$$

(No concrete texture is specified for the coupling constants: $\hat{\Gamma}, \hat{G}$ are arbitrary non-degenerate matrices with $O(1)$ elements.) The heavy fermions themselves get masses from the operators of the successively increasing dimensions:

$$\mathcal{O}_{11} = M_P 16_1^F \overline{16}_1^F, \quad \mathcal{O}_{22} = 16_2^F \tilde{A} \overline{16}_2^F, \quad \mathcal{O}_{33} = \frac{1}{M_P} (16_3^F \tilde{A}) (\tilde{A} \overline{16}_3^F) \quad (54)$$

\mathcal{O}_{33} as well as the second operator in (53) can be induced by the exchanges of some other heavy (with $M \sim M_P$) states $16^F + \overline{16}^F$, so that combinations in the parentheses transform as effective 16 and $\overline{16}$.

The choice of the A_R type 45-plet in the mixing terms (53) is of key importance. In this case the light fermions in 16_i get mass only via the mixing to the weak isosinglet F -type components in the 16_i^F states (49), while the \mathcal{F} -type (weak isodoublet) fragments are irrelevant. This maintains the $SU(4) \times SU(2) \times SU(2)'$ invariance (i.e. quark-lepton and isotopic symmetries) in the ‘heavy-to-light’ mixing terms and, interestingly, also suppresses the dangerous $d = 5$ operators causing too fast decay of the proton⁷³.

As a result, after integrating out the heavy states at the GUT scale, one obtains the Yukawa constant matrices in the following form:[§]

$$\hat{\lambda}_f^{-1} = \frac{1}{\lambda} (\hat{P}_1 + \varepsilon_f \hat{P}_2 + \varepsilon_f^2 \hat{P}_3) = \frac{1}{\lambda} \begin{pmatrix} 1 + a^2 \varepsilon_f + x^2 \varepsilon_f^2 & ab \varepsilon_f + xy \varepsilon_f^2 & xz \varepsilon_f^2 \\ ab \varepsilon_f + xy \varepsilon_f^2 & b^2 \varepsilon_f + y^2 \varepsilon_f^2 & yz \varepsilon_f^2 \\ xz \varepsilon_f^2 & yz \varepsilon_f^2 & z^2 \varepsilon_f^2 \end{pmatrix} \quad (55)$$

where $\lambda \sim (V_R/M_P)^2 \sim 10^{-5}$, $\hat{P}_{1,2,3}$ are *rank-1* matrices with $O(1)$ elements, which without loss of generality are chosen as $\hat{P}_1 = (1, 0, 0)^T \bullet (1, 0, 0)$, $\hat{P}_2 = (a, b, 0)^T \bullet (a, b, 0)$, $\hat{P}_3 = (x, y, z)^T \bullet (x, y, z)$, and $\varepsilon_f \sim \tilde{V}/M_P \sim 10^{-1} - 10^{-2}$ are small complex parameters, different for $f = u, d, e, (\nu)$. However, even if the VEV of \tilde{A} has a ‘general’ pattern, the $SO(10)$ symmetry imposes that $\varepsilon_{d,u} = \varepsilon_{15} \pm \varepsilon_3$, $\varepsilon_{e,\nu} = -3\varepsilon_{15} \pm \varepsilon_3$. Thus, only two of these four parameters are independent, and

$$\varepsilon_e = -\varepsilon_d - 2\varepsilon_u, \quad \varepsilon_\nu = 2\varepsilon_e + 3\varepsilon_u \quad (56)$$

[§] This pattern was first obtained in ref.⁴⁴ in the context of the radiative mass generation scenario.

In lowest order the Yukawa constant eigenvalues are given by diagonal entries: $\lambda_f^{(i)} \approx \lambda \varepsilon_f^{1-i}$ where $i = 1, 2, 3$ is a family index. In this way, the quark mass pattern (15),(17) is understood by means of $\varepsilon_u \ll \varepsilon_d$: $\lambda_{u,d} \sim \lambda$, $\lambda_c/\lambda_s \sim (\varepsilon_d/\varepsilon_u) \sim 10$ and $\lambda_t/\lambda_b \sim (\varepsilon_d/\varepsilon_u)^2 \sim 10^2$. This also implies that the CKM mixing emerges dominantly from the down quark matrix $\hat{\lambda}_d$: $\hat{\lambda}_u$ is much more "stretched" and essentially close to its diagonal form, so that it brings only $O(\varepsilon_u/\varepsilon_d)$ corrections to the mixing angles. Thus, at lowest order one expects that $s_{12}, s_{23} \sim \varepsilon_d$ and $s_{13} \sim \varepsilon_d^2$, which estimates are indeed good for s_{23} and s_{13} .

However, for the correct quantitative picture the matrices (55) must be diagonalized^{69,44} by assuming that $a^2 \varepsilon_f \sim 1$. Then for the Yukawa eigenvalues we obtain

$$\lambda_{u,d,e} = \frac{\lambda}{|1 + a^2 \varepsilon_{u,d,e}|}, \quad \lambda_{c,s,\mu} = \frac{\lambda |1 + a^2 \varepsilon_{u,d,e}|}{|b^2 \varepsilon_{u,d,e}|}, \quad \lambda_{t,b,\tau} = \frac{\lambda}{|z^2 \varepsilon_{u,d,e}^2|} \quad (57)$$

while the CKM angles are

$$s_{12} = \frac{|\varepsilon_d a b|}{|1 + \varepsilon_d a^2|} = \sqrt{\frac{\lambda_d}{\lambda_s} |\varepsilon_d a^2|}, \quad s_{23} = \frac{\lambda_u}{\lambda_d} \left| \frac{y z}{b^2} \varepsilon_d \right|, \quad s_{13} = \frac{\lambda_d}{\lambda_u} |x z \varepsilon_d^2| \quad (58)$$

From (57) immediately follow the relations

$$\sqrt{\frac{\lambda_b}{\lambda_t}} = \frac{\lambda_d \lambda_s}{\lambda_u \lambda_c} = \left| \frac{\varepsilon_u}{\varepsilon_d} \right|, \quad \sqrt{\frac{\lambda_b}{\lambda_\tau}} = \frac{\lambda_d \lambda_s}{\lambda_e \lambda_\mu} = \left| \frac{\varepsilon_e}{\varepsilon_d} \right| = \left| 1 + 2 \frac{\varepsilon_u}{\varepsilon_d} \right| \quad (59)$$

As soon as $|\varepsilon_u/\varepsilon_d| = \sqrt{\lambda_b/\lambda_t} \ll 1$, from (56) we obtain that $\varepsilon_d \approx -\varepsilon_e$ and hence $\lambda_b \approx \lambda_\tau$ and $\lambda_d \lambda_s \approx \lambda_e \lambda_\mu$. Thus, in spite of naive expectation, the *Grand Prix* of $b - \tau$ unification is not lost: it is more precise the more $t - b$ are split. This is a direct result of the $SO(10)$ symmetry relation (56). Otherwise, since $\lambda_{b,\tau}$ emerge at $O(\varepsilon^2)$ level, e.g. the factor of 2 difference among ε_d and ε_e would cause already factor of 4 splitting between λ_b and λ_τ .

On the other hand, the experimental value of the Cabibbo angle $s_{12} \simeq (m_d/m_s)^{1/2}$ implies that $|\varepsilon_d a^2| \simeq 1$. Then, owing to the same relation $\varepsilon_d \approx -\varepsilon_e$, the constants $\lambda_d = \lambda |1 + \varepsilon_d a^2|^{-1}$ and $\lambda_e = \lambda |1 + \varepsilon_e a^2|^{-1}$ should deviate to different sides from $\lambda_u = \lambda$ by a factor of 2 or so. Hence, the first family can indeed play a role of the *Yukawa unification* point, with its splitting understood by the same mechanism that enhances the Cabibbo angle up to the value $s_{12} \simeq \sqrt{m_d/m_s}$. The other mixing angles in (58) stay much smaller: $s_{23} \sim \varepsilon_d$ and $s_{13} \sim \varepsilon_d^2$, in accord to the observed pattern (19).

For more details of the model one can address refs.⁶⁹, where the detailed numerical analysis was carried out. In particular, the masses of leptons and c and b quarks, the ratio m_s/m_d and the value of s_{12} were taken as input parameters and the u, d, s quark masses, top mass and $\tan \beta$ were computed: $m_s \simeq 150$ MeV, $m_d \simeq 7$ MeV and $m_u/m_d \simeq 0.5 - 0.7$. The top mass emerges in infrared-fixed regime ($\lambda_t \sim 1.5$ at GUT scale), and thus it is naturally in the 100 GeV range. However, there emerges an upper limit $\tan \beta < 1.7$, which translates into the upper bound $M_t < 165$ GeV.

Zero textures of eq. (51) give a bigger amount of predictions than the ansatz of eq. (55), as far as the model⁷³ is based on weaker assumptions than the models of ref.⁶⁶. However, predictive power of the former approach can be enhanced further e.g. by imposing specific zero textures on the coupling constants in (53) which in general analysis were left arbitrary.

4. GIFT for Fermion Masses: SUSY $SU(6)$ Model

4.1. Higgs Doublets as Pseudo-Goldstone Bosons

SUSY $SU(6)$ model⁷⁰ (see also^{71,72,73,74}) was originally designed for the natural solution to the gauge hierarchy and doublet-triplet (DT) splitting problems via the elegant GIFT (*Goldstones Instead of Fine Tuning*) mechanism.[¶] The $SU(6)$ model⁷⁰ is a minimal extension of $SU(5)$: the Higgs sector contains supermultiplets Σ and $H + \bar{H}$ respectively in adjoint 35 and fundamental $6 + \bar{6}$ representations, in analogy to 24 and $5 + \bar{5}$ of $SU(5)$. However, this model drastically differs from the other GUTs where the Higgs sector usually consists of two different sets: one is for the GUT symmetry breaking (e.g. 24-plet in $SU(5)$), while another containing the Higgs doublets (like $5 + \bar{5}$ in $SU(5)$) is just for the electroweak symmetry breaking. The $SU(6)$ theory has no special superfields for the second purpose: 35 and $6 + \bar{6}$ constitute a minimal Higgs content needed for the local $SU(6)$ symmetry breaking down to $SU(3) \times SU(2) \times U(1)$. As for the MSSM Higgs doublets $\phi_{1,2}$, they emerge from Σ and H, \bar{H} , as Goldstone modes of the accidental global symmetry $SU(6)_\Sigma \times U(6)_H$. This global symmetry arises if mixing terms of the form $\bar{H}\Sigma H$ are suppressed in the Higgs superpotential⁷⁰. Then $\phi_{1,2}$ being strictly massless in the exact SUSY limit, acquire non-zero mass terms (interestingly, including also the μ -term) due to the spontaneous SUSY breaking.

Once the Higgs doublets emerge as Goldstone modes, their couplings to fermions have peculiarities leading to new possibilities towards understanding of the flavour structure. Indeed, should the Yukawa terms also respect the $SU(6)_\Sigma \times U(6)_H$ global symmetry, then $\phi_{1,2}$ being the Goldstone modes would have the *vanishing* Yukawa couplings to all fermions which remain massless after the GUT symmetry breaking down to the MSSM, that are ordinary quarks and leptons. Thus, the couplings relevant for fermion masses have to *explicitly* violate $SU(6)_\Sigma \times U(6)_H$. This constraint leads to striking possibilities to understand the fermion mass and mixing pattern even in completely ‘democratic’ approach, without invoking the the horizontal symmetry arguments. In particular, it was shown in ref.⁷² that *only* the top quark can get ~ 100 GeV mass through the renormalizable Yukawa coupling, while other fermion masses can emerge only through the higher order operators and thus are suppressed by powers of the Planck scale M_P .

In order to built a consistent GIFT model, one has to find some valid symmetry reasons to forbid the mixing terms like $\bar{H}\Sigma H$: otherwise the theory has no accidental

[¶]The GIFT mechanism for the DT splitting was first suggested in the context of SUSY $SU(5)$ by assuming an *ad hoc* $SU(6)$ global symmetry of the Higgs superpotential^{75,76}. Results for fermion masses, however, are specific of the gauged $SU(6)$ theory.

global symmetry. It is natural to use for this purpose the discrete symmetries, which in principle could emerge in the string theory context. In addition, they can provide a proper pattern of the higher order operators inducing the fermion masses. Possible consistent models were suggested in refs.^{73,74}. Below we consider the $SU(6)$ model of ref.⁷³ with the flavour-blind discrete symmetry Z_3 .

4.2. $SU(6) \times Z_3$ Model

We assume that below the Planck scale M_P the field theory is given by SUSY GUT with the $SU(6)$ gauge symmetry, equipped with the flavour-blind discrete symmetries $Z_3 \times Z_2$. Z_2 stands for the usual matter parity, under which the fermion superfields change the sign while the Higgs ones stay invariant. Z_2 parity is known to be free of discrete anomalies⁷⁷, and is needed for suppressing the B and L violating $d = 4$ operators. The theory contains the following chiral superfields:

(i) Higgs sector: vectorlike set of supermultiplets $\Sigma_1(35)$, $\Sigma_2(35)$, $H(6)$, $\bar{H}(\bar{6})$ and singlet Y ;

(ii) Fermion sector: chiral, anomaly free supermultiplets $(\bar{6} + \bar{6}')_i$, 15_i ($i = 1, 2, 3$ is a family index) and 20;

(iii) F -fermion sector: heavy vector-like matter multiplets like $15_F + \bar{15}_F$, etc. with large ($\sim M_P$) $SU(6)$ invariant mass terms. They will be needed for the light fermion mass generation through the HFE mechanism^{48,49,50,51}.

The field content of the model and their Z_3 charges are given in Table 1. Z_3 symmetry satisfies the anomaly cancellation constraints⁷⁷ and it can be regarded as a gauge discrete symmetry.

In the following we assume that all coupling constants in the Higgs as well as in the Yukawa sectors are of the order of 1. (For comparison, the gauge coupling constant at the GUT scale is $g_X \simeq 0.7$.) The most general renormalizable Higgs superpotential compatible with the $SU(6) \times Z_3$ symmetry is

$$\mathcal{W} = M_\Sigma \Sigma_1 \Sigma_2 + \lambda_1 \Sigma_1^3 + \lambda_2 \Sigma_2^3 + \lambda \Sigma_1 \Sigma_2 + M_H \bar{H} H + \rho Y (\bar{H} H - \Lambda^2) + M_Y Y^2 + \xi Y^3 \quad (60)$$

and it has an accidental global symmetry $SU(6)_\Sigma \times U(6)_H$, related to independent transformations of Σ and H .^{||} In the limit of exact SUSY (i.e. of vanishing F and D terms), among the other degenerated vacua, there is the following one:

$$\langle \Sigma_{1,2} \rangle = V_{1,2} \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}, \quad \langle H \rangle = \langle \bar{H} \rangle = V_H \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \langle Y \rangle = V_Y \quad (61)$$

^{||} Notice that global $SU(6)_\Sigma \times U(6)_H$ is not a symmetry of the whole Lagrangian: the Yukawa and the gauge couplings (D -terms) do not respect it. However, in the exact supersymmetry limit it is effective for the field configurations on the vacuum valley where $D = 0$. Owing to non-renormalization theorem, it cannot be spoiled by the radiative corrections.

Z_3 :	Higgs	fermions	F -fermions
ω	Σ_1	$\bar{6}_i, \bar{6}'_i, 20$	$\bar{15}_F^2, \bar{15}_F^3, 20_F, 35_F, \bar{70}_F, 84_F$
$\bar{\omega}$	Σ_2	15_i	$15_F^2, 15_F^3, 20_F, 35_F, 70_F, 84_F$
$inv.$	H, \bar{H}, Y	$-$	$\bar{15}_F^1, 15_F^1, 20_F^{1,2}, \bar{105}_F, 105_F, \bar{210}_F, 210_F$

Table 1: Z_3 -transformations of various $SU(6)$ supermultiplets ($\omega = \exp(i2\pi/3)$).

where, provided that $\Lambda \gg V_\Sigma = (V_1^2 + V_2^2)^{\frac{1}{2}}$, we have:

$$V_Y = \frac{M_H}{\rho}, \quad V_{1,2} = \frac{M_\Sigma + \lambda V_Y}{(\lambda_1 \lambda_2)^{\frac{1}{3}} \lambda_{1,2}^{\frac{1}{3}}}, \quad V_H = \Lambda + O\left(\frac{V_\Sigma^2}{\Lambda}\right) \quad (62)$$

After SUSY breaking, the configuration (61) can indeed be a true vacuum state for a proper range of the soft parameters. Then H, \bar{H} break $SU(6)$ down to $SU(5)$ while $\Sigma_{1,2}$ break $SU(6)$ down to $SU(4) \times SU(2) \times U(1)$, and both channels together lead to the local symmetry breaking down to $SU(3) \times SU(2) \times U(1)$. At the same time, the global symmetry $SU(6)_\Sigma \times U(6)_H$ is broken down to $[SU(4) \times SU(2) \times U(1)]_\Sigma \times U(5)_H$. Most of the Goldstone degrees correspond to generators of the broken local $SU(6)$ and they are eaten up by the $SU(6)$ gauge superfields through the Higgs mechanism. However, since the global symmetry of the ground state exceeds the global one, a couple of fragments survive and present in particle spectrum at lower energies as the Goldstone superfields. These constitute the MSSM Higgs doublets $\phi_{1,2}$ which in terms of the doublet (anti-doublet) fragments in $\Sigma_{1,2}$ and H, \bar{H} are given as

$$\phi_2 = c_\eta(c_\sigma \phi_{\Sigma_1} + s_\sigma \phi_{\Sigma_2}) - s_\eta \phi_H, \quad \phi_1 = c_\eta(c_\sigma \bar{\phi}_{\Sigma_1} + s_\sigma \bar{\phi}_{\Sigma_2}) - s_\eta \bar{\phi}_{\bar{H}} \quad (63)$$

where $\tan \eta = 3V_\Sigma/V_H$ and $\tan \sigma = V_2/V_1 = (\lambda_1/\lambda_2)^{\frac{1}{3}} \sim 1$. In the following we assume that $M_P \gg V_H \gg V_\Sigma \simeq M_X \simeq 10^{16}$ GeV, as it is motivated by the $SU(5)$ unification of the gauge couplings. In this case the doublets $\phi_{1,2}$ dominantly come from $\Sigma_{1,2}$ while in H, \bar{H} they are contained with small weight $\sim 3V_\Sigma/V_H$.

The scalar fields in $\phi_{1,2}$ then get mass from the soft SUSY breaking terms⁷⁸:

$$V_{SB} = Am_S \mathcal{W}_3 + Bm_S \mathcal{W}_2 + m_S^2 \sum_k |\varphi_k|^2, \quad (64)$$

where φ_k imply all scalar fields involved, $\mathcal{W}_{3,2}$ respectively are trilinear and bilinear terms in (60) and A, B, m_S are the soft breaking parameters. SUSY breaking relaxes radiative corrections which lift the vacuum degeneracy (mainly due to the large top Yukawa coupling, origin of which we will clarify below) and fix the VEVs v_1 and v_2 . The effects of radiative corrections leading to the electroweak symmetry breaking were studied recently in refs.⁷⁹.

Interestingly, the GIFT scenario naturally solves also the μ -problem. Taking into account the soft SUSY breaking terms (64) in minimization of the Higgs potential of Σ and H, \bar{H} , one observes that the VEVs $V_{1,2}$ are shifted by an amount of $\sim m_S$

as compared to the ones of eq. (62) calculated in the exact SUSY limit. Then substituting these VEVs back in superpotential, this shift gives rise to the $\mu\phi_1\phi_2$ term, with $\mu \sim m_S$. Thus, in GIFT scenario the (supersymmetric) μ -term emerges as a consequence of the SUSY breaking.

Thus, the $SU(6)$ model naturally solves both the DT splitting and the μ problems. The Higgs doublets $\phi_{1,2}$ remain light, $\mu \sim m_S$, and their triplet partners are superheavy: the triplets from $\Sigma_{1,2}$ have masses $\sim M_X$, while the ‘Goldstone’ triplets from H, \bar{H} acquire masses $\sim V_H$ due to the Higgs mechanism by mixing with the $SU(6)$ gauge superfields.

4.3. Fermion Masses and Mixing in $SU(6) \times Z_3$ Model

Now we show how the observed hierarchy of fermion masses and mixings can be naturally explained in terms of small ratios $\varepsilon_\Sigma = V_\Sigma/V_H$ and $\varepsilon_H = V_H/M_P$.

The most general Yukawa superpotential allowed by the $SU(6) \times Z_3$ symmetry is

$$\mathcal{W}_{Yuk} = G 20\Sigma_1 20 + \Gamma 20H15_3 + \Gamma_{ij}15_i\bar{H}\bar{6}'_j, \quad i, j = 1, 2, 3 \quad (65)$$

where all coupling constants are $O(1)$. Without loss of generality, one can always redefine the basis of 15-plets so that only the 15_3 state couples 20-plet in (65). Also, among six $\bar{6}$ -plets one can always choose three of them (denoted in eq. (65) as $\bar{6}'_{1,2,3}$) which couple $15_{1,2,3}$ while the other three states $\bar{6}_{1,2,3}$ have no Yukawa couplings.

Already at the scale V_H of the gauge symmetry breaking $SU(6) \rightarrow SU(5)$ the fermion content reduces to the one of the minimal $SU(5)$. Indeed, in terms of the $SU(5)$ subgroup the fermions under consideration read as

$$\begin{aligned} 20 &= 10 + \bar{10} = (q + u^c + e^c)_{10} + (Q^c + U + E)_{\bar{10}} \\ 15_i &= (10 + 5)_i = (q_i + u_i^c + e_i^c)_{10} + (D_i + L_i^c)_5 \\ \bar{6}_i &= (\bar{5} + 1)_i = (d_i^c + l_i)_{\bar{5}} + n_i \\ \bar{6}'_i &= (\bar{5} + 1)'_i = (D_i^c + L_i)_{\bar{5}'} + n'_i, \quad i = 1, 2, 3 \end{aligned} \quad (66)$$

In the spirit of survival hypothesis⁵², the extra vector-like fermions $\bar{10} + 10_3$ and $(5 + \bar{5}')_{1,2,3}$, get $\sim V_H$ masses from couplings (65):

$$\Gamma V_H \bar{10} 10_3 + \Gamma_{ij} V_H 5_i \bar{5}'_j + G V_1 (U u^c - 2E e^c), \quad (67)$$

and thereby decouple from the light states which remain as $\bar{5}_{1,2,3}$, $10_{1,2}$, 10 and singlets n_i, n'_i (we neglect $\sim \varepsilon_\Sigma$ mixing between the $u^c - u_3^c$ and $e^c - e_3^c$ states).

The couplings of 20-plet in (65) explicitly violate the global $SU(6)_\Sigma \times U(6)_H$ symmetry. Hence, the up-type quark from 20 (to be identified as top) has *non-vanishing* coupling with the Higgs doublet ϕ_2 . As far as $V_H \gg V_\Sigma$, it essentially emerges from $G 20\Sigma_1 20 \rightarrow G qu^c \phi_2$. Thus, *only* the top quark can have ~ 100 GeV mass due to the large Yukawa constant $\lambda_t = G \sim 1$.

Other fermions would stay massless unless we invoke the HOPs scaled by inverse powers of M_P , which will emerge by integrating out the F -fermions. Before addressing the concrete HFE scheme, let us first analyse the general structure of the

possible HOPs. Obviously, Z_3 symmetry forbids the $d = 5$ ‘Yukawa’ terms in the superpotential. However, the following $d = 6$ and $d = 7$ operators are allowed:**

$$\begin{aligned}
d = 6 : \quad \mathcal{B} &= \frac{B}{M_P^2} 20\bar{H}(\Sigma_1\bar{H})\bar{6}_3, \quad \mathcal{C} = \frac{C_{ij}}{M_P^2} 15_i H(\Sigma_2 H) 15_j \\
\mathcal{S} &= \frac{S_{ik}^{(1)}}{M_P^2} 15_i(\Sigma_1\Sigma_2\bar{H})\bar{6}_k + \frac{S_{ik}^{(2)}}{M_P^2} 15_i(\Sigma_1\bar{H})(\Sigma_2\bar{6}_k) \\
\mathcal{N} &= \frac{N_{kl}}{M_P^2} \bar{6}_k H(\Sigma_1 H)\bar{6}_l
\end{aligned} \tag{68}$$

$$\begin{aligned}
d = 7 : \quad \mathcal{D} &= \frac{D_{ik}^{(1)}}{M_P^3} 15_i(\Sigma_1^3\bar{H})\bar{6}_k + \frac{D_{ik}^{(2)}}{M_P^3} 15_i(\Sigma_1^2\bar{H})(\Sigma_1\bar{6}_k) + \\
&\frac{D_{ik}^{(3)}}{M_P^3} 15_i(\Sigma_1\bar{H})(\Sigma_1^2\bar{6}_k) + \frac{D_{ik}^{(4)}}{M_P^3} 15_i(\Sigma_1\bar{H})\bar{6}_k \cdot \text{Tr}\Sigma_1^2 \\
\mathcal{U} &= \frac{U_{ij}^{(1)}}{M_P^3} 15_i H(\Sigma_1^2 H) 15_j + \frac{U_{ij}^{(2)}}{M_P^3} 15_i H(\Sigma_1 H)\Sigma_1 15_j
\end{aligned} \tag{69}$$

($SU(6)$ indices are always contractrd so that combinations in the parentheses transform as effective $\bar{6}$ or 6). Since the $\bar{6}'$ and 15_3 states already have $\sim V_H$ masses, these operators are relevant only for the light states in 20 , $15_{1,2}$ and $\bar{6}_{1,2,3}$. One can always redefine the basis of $\bar{6}$ -plets so that only the $\bar{6}_3$ couples 20 in eq. (68). In addition, we assume that constants B , C_{ij} , etc. all are order 1 as well as the constants in (65).

Operator \mathcal{B} gives rise to the b quark and τ lepton masses. At the MSSM level it reduces to the Yukawa couplings $\varepsilon_H^2 B(qd_3^c + e^c l_3)\phi_1$. Hence, though b and τ belong to the 20 -plet as well as t , their Yukawa constants are by factor $\sim \varepsilon_H^2$ smaller than λ_t . In addition, the $b - \tau$ Yukawa constants are automatically unified at the GUT scale: $\lambda_b = \lambda_\tau$, up to $\sim \varepsilon_\Sigma^2$ corrections due to the mixing of e^c and e_3^c states in eq. (67).

Operator \mathcal{C} contributes the up quark Yukawa constants of the first and second families, as $\lambda_{ij}^u = \varepsilon_H^2 C_{ij}$ ($i, j = 1, 2$). As for the operators \mathcal{S} and \mathcal{D} , they induce the Yukawa constants of the down quarks and charged leptons respectively as

$$\begin{aligned}
\lambda_{ik}^d &= \varepsilon_\Sigma \varepsilon_H^2 (S_{ik}^{(1)} - S_{ik}^{(2)}), & \tilde{\lambda}_{ik}^d &= \varepsilon_\Sigma^2 \varepsilon_H^3 (3D_{ik}^{(1)} - D_{ik}^{(2)} + D_{ik}^{(3)} + 12D_{ik}^{(4)}) \\
\lambda_{ik}^e &= \varepsilon_\Sigma \varepsilon_H^2 (S_{ik}^{(1)} + 2S_{ik}^{(2)}), & \tilde{\lambda}_{ik}^e &= \varepsilon_\Sigma^2 \varepsilon_H^3 (3D_{ik}^{(1)} + 2D_{ik}^{(2)} + 4D_{ik}^{(3)} + 12D_{ik}^{(4)})
\end{aligned} \tag{70}$$

($i = 1, 2$, $k = 1, 2, 3$). Clearly, the mass hierarchy between the first and second families fermions can have a realistic shape only if the latter emerge from the $d = 6$ operators \mathcal{C} and \mathcal{S} , while the first family get masses from the $d = 7$ operators \mathcal{U} and \mathcal{D} . Certainly, this can be done by introducing some horizontal symmetry which could fix the mass matrix textures and the structure of the HOPs involved in mass generation (see e.g. ref.⁷⁴). However, it is interesting that in the context of the HFE mechanism the basic explanation of the fermion mass and mixing pattern can be achieved in a completely ‘democratic’ approach, without appealing to any horizontal symmetry. This can be obtained as a result of the properly chosen representations

**The terms like $15\bar{H}(\Sigma_1\Sigma_2\bar{6})$ or $15\bar{H}\bar{6}\cdot\text{Tr}(\Sigma_1\Sigma_2)$ do not violate the global $SU(6)_\Sigma \times U(6)_H$ symmetry and are therefore irrelevant.

for F -fermions. In particular, the HFE can induce the HOPs (68) in such a manner that C_{ij} and $S_{ik}^{1,2}$ emerge as the *rank-1* matrices, which then without loss of generality can be chosen as

$$C_{ij} = \begin{pmatrix} 0 & 0 \\ 0 & C \end{pmatrix}, \quad S_{ik}^{(1,2)} \propto \begin{pmatrix} 0 & s_\theta S_2 & s_\theta S_3 \\ 0 & c_\theta S_2 & c_\theta S_3 \end{pmatrix} \quad (71)$$

In addition, such HOPs should provide definite Clebsch structures, which would allow to obtain certain mass relations.

The relevant HFE's involving the F -fermions of Table 1 are shown in Figs. 1-3. As a result, one obtains the following pattern of the Yukawa couplings at the GUT scale M_X :

$$\begin{matrix} & u_1^c & u_2^c & u^c \\ q_1 & 0 & \varepsilon_\Sigma \varepsilon_H^3 U & 0 \\ q_2 & \varepsilon_\Sigma \varepsilon_H^3 U' & \varepsilon_H^2 C & 0 \\ q & 0 & 0 & G \end{matrix} \cdot \phi_2 \quad (72a)$$

$$\begin{matrix} & d_1^c & d_2^c & d_3^c \\ q'_1 & J \varepsilon_\Sigma^2 \varepsilon_H^3 D_1 & J \varepsilon_\Sigma^2 \varepsilon_H^3 D_2 & J \varepsilon_\Sigma^2 \varepsilon_H^3 D_3 \\ q'_2 & J \varepsilon_\Sigma^2 \varepsilon_H^3 D'_2 & K \varepsilon_\Sigma \varepsilon_H^2 S_2 & K \varepsilon_\Sigma \varepsilon_H^2 S_3 \\ q & 0 & 0 & \varepsilon_H^2 B \end{matrix} \cdot \phi_1 \quad (72b)$$

$$\begin{matrix} & l_1 & l_2 & l_3 \\ e_1^{lc} & \varepsilon_\Sigma^2 \varepsilon_H^3 D_1 & \varepsilon_\Sigma^2 \varepsilon_H^3 D_2 & \varepsilon_\Sigma^2 \varepsilon_H^3 D_3 \\ e_2^{lc} & \varepsilon_\Sigma^2 \varepsilon_H^3 D'_2 & \varepsilon_\Sigma \varepsilon_H^2 S_2 & \varepsilon_\Sigma \varepsilon_H^2 S_3 \\ e^c & 0 & 0 & \varepsilon_H^2 B \end{matrix} \cdot \phi_1 \quad (72c)$$

Notice that the basis of down quarks (in $15'_{1,2}$) is already ‘Cabibbo’ rotated with respect to the upper quarks basis $15_{1,2}$, by the angle $\theta \sim 1$ (see eq. (71)). The HFE shown in Figs. 1,3 induce operators \mathcal{S} and \mathcal{D} respectively in combinations $\mathcal{S} \propto \mathcal{S}_1 + 2\mathcal{S}_2$ and $\mathcal{D} \propto \mathcal{D}_1 + \mathcal{D}_3 - \mathcal{D}_4$ (in terms of the possible operators in (68) and (69)). Then the Clebsch factors are fixed as $J = 8/5$ and $K = -1/5$.

From (72) we obtain the Yukawa coupling eigenvalues at the GUT scale

$$\begin{aligned} 3^{rd} \text{ family :} & \quad \lambda_t \sim 1, \quad \lambda_\tau = \lambda_b \sim \varepsilon_H^2 \\ 2^{nd} \text{ family :} & \quad \lambda_c \sim \varepsilon_H^2, \quad \lambda_\mu = 5\lambda_s \sim \varepsilon_\Sigma \varepsilon_H^2 \\ 1^{st} \text{ family :} & \quad \lambda_u \sim \varepsilon_\Sigma^2 \varepsilon_H^4, \quad \lambda_e = \frac{5}{8}\lambda_d \sim \varepsilon_\Sigma^2 \varepsilon_H^3 \end{aligned} \quad (73)$$

while the CKM angles are (notice different parametrization from that of eq. (6)):

$$V_{\text{CKM}} \approx \begin{pmatrix} 1 & s_{12} & s_{12}s_{23} - s_{13}e^{-i\delta} \\ -s_{12} & 1 & s_{23} + s_{12}s_{13}e^{-i\delta} \\ s_{13}e^{i\delta} & -s_{23} & 1 \end{pmatrix}, \quad s_{12}(\theta) \sim 1, \quad s_{23} \sim \frac{\lambda_s}{\lambda_b}, \quad s_{13} \sim \frac{\lambda_d}{\lambda_b} \quad (74)$$

where the CP-phase δ arises due to the complex Yukawa constants in the theory.

Taking into account the RG running for the Yukawa constants (22), this pattern can be confronted to the low energy (experimental) observables (7) and (9). We see that in the context of small $\tan \beta$ (which is rather natural in the GIFT scenario), eqs. (73) and (74) explain all basic features of the fermion masses and mixings in terms of small parameters $\varepsilon_H, \varepsilon_\Sigma \sim 0.1$ (compare e.g. with pattern of eqs. (14) and (19)). Moreover, from (73) follows that

$$\frac{m_d}{m_s} \simeq 8 \frac{m_e}{m_\mu} \approx \frac{1}{25} [1 + O(\varepsilon_\Sigma)] \quad (75)$$

($\sim \varepsilon_\Sigma$ correction can arise due to mixing terms in (72)), while for the s quark running mass at $\mu = 1$ GeV we obtain $m_s = (\eta A_d/5A_e)m_\mu = 100 - 150$ MeV.

As for the neutrinos, they acquire small Majorana mass from the operator \mathcal{N} in (68) induced by the HFE in Fig. 2:

$$m_{ij}^\nu \sim N_{ij} \frac{\varepsilon_H v^2}{\varepsilon_\Sigma M_P} \quad (76)$$

which for $\varepsilon_\Sigma, \varepsilon_H \sim 0.1$ gives $m_\nu \sim v^2/M_P \sim 10^{-5}$ eV. Since our ‘democratic’ approach in general implies no hierarchy in constants $N_{kl} \sim 1$, one expects that neutrino mixing angles are $O(1)$. Thus the predictions for the neutrino oscillation parameters are in the range (11), needed for the “just-so” solution to the solar neutrino problem¹⁰.

Let us remark that the above results are obtained from the general operator analysis of all possible HFE, and the F -fermion content of the Table 1 is uniquely selected among several other possibilities⁷³. In constructing the HOPs the following constraints have been taken into account:

(A) In order to ensure that the $d = 6$ operators \mathcal{C}, \mathcal{S} induce only the second family fermion (c, s, μ) masses, they have to be induced by the unique exchange chain.

(B) Once the HFE generating \mathcal{C} and \mathcal{S} are selected, the $d = 7$ operators \mathcal{D} and \mathcal{U} should be constructed by the F -fermion exchange chains which are irreducible to $d = 6$ operators: otherwise the mass hierarchy between the first and second families would be spoiled. In other words, the exchange chains should not allow to replace $\Sigma_1 \times \Sigma_1$ by Σ_2 , so that the (symmetric) tensor product $\Sigma_1 \times \Sigma_1$ should effectively act as the 189 or 405 representations of $SU(6)$. This condition requires the large representations like 105, 210, etc. to be involved into the game.

All possible HFE satisfying the conditions (A) and (B) have been classified⁷³. It was shown, that operator \mathcal{D} can be induced only by few other irreducible chains involving large representations, which give rise to the combinations $\mathcal{D}_1 - \mathcal{D}_2 + \mathcal{D}_3 + \mathcal{D}_4$: ($J = 1$), $\mathcal{D}_1 \mp \mathcal{D}_4$: ($J = 1$), $\mathcal{D}_1 - 2\mathcal{D}_2 - \mathcal{D}_4$: ($J = 11/17$), and $\mathcal{D}_1 + \mathcal{D}_3 - \mathcal{D}_4$: ($J = 8/5$). Therefore, the HFE of Fig. 3 implying $J = 8/5$ is selected as the only one feasible choice: all other possibilities lead to $\lambda_d \leq \lambda_e$ and are thus unacceptable.

Also the HFE relevant for operator \mathcal{S} have been classified. By scanning all possible representations for the F -fermions, it has been obtained⁷³ that \mathcal{S} can emerge only in the combinations \mathcal{S}_1 : ($K = 1$), \mathcal{S}_2 : ($K = -1/2$), $\mathcal{S}_1 \pm \mathcal{S}_2$: ($K = 0, -2$ respectively), $\mathcal{S}_1 - 2\mathcal{S}_2$: ($K = -1$), and $\mathcal{S}_1 + 2\mathcal{S}_2$: ($K = -1/5$). We have chosen the latter case

uniquely selected by the HFE in Fig. 1. All other cases are unacceptable: $K = 0$ ($|K| \geq 1$) leads to massless (too heavy) s quark, while $K = -1/2$ in combination with $J = 8/5$ implies $m_d/m_s \approx 1/70$.

As for the operators \mathcal{C} and \mathcal{U} , the only possible HFE obeying conditions (A) and (B) are the ones shown in Figs. 1,3. Thus, among all possible F -fermions only the ones selected in Table 1 lead to acceptable pattern of the HOPs in (68) and (69).

Concluding, the fermion mass and mixing pattern can be naturally explained in our scheme without appealing to any horizontal symmetry, provided that the scales M_P , V_H and V_Σ are related as $V_\Sigma/V_H \sim V_H/M_P \sim 0.1$. As far as the scale $V_\Sigma \simeq 10^{16}$ GeV is fixed by the $SU(5)$ unification of the gauge couplings, these relations in turn imply that $V_H \sim 10^{17}$ GeV and $M_P \sim 10^{18}$ GeV, so that M_P is indeed close to the string or Planck scale. It is also noteworthy also that the scale $V_H \sim \sqrt{M_X M_P}$ could naturally emerge in the context of the models discussed in refs.^{73,74}.

5. Conclusion

In conclusion, I briefly discuss some actualities of the flavour problem:

- *How many families?* This difficult question is originated by the fact of family replication itself. There is no simple answer to the question “why three families?”, or “why *only* three families?”. In the SM (MSSM) as well as in GUTs the number of families N_f remains to be an arbitrary parameter. Nevertheless, it seems that Nature prefers 3-family variant. The measurement of the Z -boson decay width indicates that there are no more standard-like families with light neutrinos. The standard model precision data constrain the number of families as $N_f < 6$, so that only one or two extra heavy families with heavy ($M > M_Z/2$) neutrinos are still allowed.

In the MSSM with very modest values of $\tan \beta$, there is still some (though not much) room for the fourth family⁸⁰. Concerning SUSY GUTs, the presence of the fourth family would not affect the gauge coupling unification. However, this would affect the RG running of the Yukawa constants and thus spoil the *Grand Prix* of the $b - \tau$ unification. In the SUSY $SU(6)$ model there is no room for extra heavy family since due to Goldstone nature of the Higgs doublets only one fermion, namely the top, can have ~ 100 GeV mass.

- *Neutrino Masses.* If the $SU(5)$ is a fundamental theory up to the Planck scale M_P , or if it is embedded in $SU(6)$ at some scale below M_P , then the natural value of the neutrino masses is rather $\hat{m} = v^2/M_P \sim 10^{-5}$ eV. This can solve the SNP via ‘just-so’ oscillation¹⁰, but other neutrino hints^{12,13,14} are left unexplained. As for the $SO(10)$ theory, it contains the RH neutrinos ν^c , and generates the LH neutrino masses by means of the seesaw mechanism³. The predictive $SO(10)$ frameworks^{66,67,73} allow to calculate the neutrino Dirac masses, and their spectrum typically has the same shape as that of the charged fermions (see e.g. eqs. (51) or (55)). However, this cannot suffice for deducing the neutrino mass pattern, since now the question mark is contained in the RH neutrinos Majorana masses M_R . The latter can emerge through

the $d = 5$ operators involving the the Higgs 16-plets $\psi, \bar{\psi}$:

$$\frac{g_{ij}}{M_P}(16_i\bar{\psi})(\bar{\psi}16_j) \quad (77)$$

Thus we obtain $M_R \sim M_{10}^2/M_P$, where M_{10} is the $SO(10) \rightarrow SU(5)$ breaking scale which can range from $M_X \simeq 10^{16}$ GeV to $M_P = 10^{19}$ GeV, and therefore $M_R \sim 10^{12-16}$ GeV. This corresponds to the mass of the heaviest neutrino (ν_τ) of about $10^{-3} - 10$ eV, in which case the SNP can be due to the MSW mechanism¹¹, and still some room can be left for explaining one of the other neutrino hints. For example, if $m_{\nu_\tau} \sim 0.1$ eV and $m_{\nu_\mu} \sim 10^{-2.5}$ eV, the SNP can be explained via the MSW oscillation $\nu_e \rightarrow \nu_\mu$ while the ANP can be explained via the $\nu_\mu \rightarrow \nu_\tau$ oscillation with large mixing angle. More precise predictions, however, would require to fix the pattern of constants g_{ij} by imposing some symmetry constraints.

It is worth to remark also that if all recent neutrino hints^{9,12,13,14} will be confirmed by future experiments, then three neutrino states $\nu_{e,\mu,\tau}$ will not suffice for thir explanation. It was shown in ref.⁸¹ that only one possibility is compatible with all these data, which requires an extra light sterile neutrino ν_s beyond the three known neutrinos. Then, assuming that $m_{\nu_{e,s}} \ll m_{\nu_{\mu,\tau}}$, the SNP can be explained by the $\nu_e \rightarrow \nu_s$ oscillation and the ANP by the $\nu_\mu \rightarrow \nu_\tau$ oscillation. In addition, almost degenerate ν_μ and ν_τ with mass of about 2.5 eV provide the cosmological HDM and can also explain the LSND oscillation $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$.

Of course, one needs to explain naturally the existence of so light sterile neutrino. According to the recent proposals, it could emerge in the particle spectrum as a light pseudo-Goldstone fermion⁸², or as a neutrino of a hidden ‘mirror’ world⁸³.

- *Proton decay.* A realistic SUSY GUT must be capable to prevent proton from too fast decay. The gauge ‘dinosaur’ boson mediated proton decays are not dangerous in SUSY GUT owing to the large unification scale $M_X \sim 10^{16}$ GeV. In SUSY GUTs proton wants to decay rather fastly via the $d = 5$ operators³⁰:

$$O_L = \frac{1}{M} q_i q_j q_k l_m, \quad O_R = \frac{1}{M} u_i^c u_j^c d_k^c e_m^c \quad (78)$$

which are mediated by exchanges of the heavy triplets T, \bar{T} with mass $M \sim M_X$. These couple to fermions as $q_i q_j T, q_k l_m \bar{T}$ etc., which couplings in SUSY GUT emerge on the same grounds as the ones of the Higgs doublets $\phi_{1,2}$ in eq. (21). Although operators (78) involve the small Yukawa constants and mixing angles, they are anyway dangerous since they contain only one power of the large scale M_X . The operators O_L dressed by Winos bring dominant contribution to the proton decay via the channel $p \rightarrow K\nu$. A dedicated analysis⁸⁴ in the minimal $SU(5)$ model shows that for a typical parameter range proton lifetime in the above mode is about 2-3 orders of magnitude below the present experimental bound, and can be marginally reconciled with this bound if all inherent uncertainties are taken at extreme borders. In generic GUTs with nontrivial Clebsches the prediction of minimal $SU(5)$ are not valid. However, if there are no conspiracies, the problem still remains since the typical Clebsches about 2 or 3 in general do not suffice to properly suppress proton decay.

However, there are several theoretical possibilities for natural solution of this problem. The proton decay can be suppressed by the same mechanism which provides the predictivity for fermion masses. This happens, e.g. in the ‘inverse’ hierarchy $SO(10)$ model⁶⁹ where the dangerous operators O_L vanish automatically, while the more safe operators O_R are left with a chance to show up in future experiments, essentially via the decay modes like $p \rightarrow K\mu$, etc. There also exist more devoted models in which both operators $O_{L,R}$ can be killed by special arrangements in the Higgs or Fermion sectors^{64,85,86}.

Proton decay can be (at least partially) suppressed by the horizontal symmetry. For example, in the $SU(5) \times SU(3)_H$ model⁵⁰ the light up quarks (u and c) get mass via antisymmetric operators $\frac{1}{M} 10_\alpha \chi^{[\alpha\beta]} H 10_\beta$ ($\alpha, \beta = 1, 2, 3$ is a family index). On the other hand, the coupling $q_\alpha q_\beta T$ is symmetric, and thus it cannot emerge from antisymmetric operator. Then only $q_3 q_3 T$ is allowed, which emerges from the symmetric operator in (41), so that only the third family sfermions contribute the dominant decay mode $p \rightarrow K\nu$. This can suppress the proton decay rate by about 2 orders of magnitude as compared to the minimal SUSY $SU(5)$ prediction.

- *Sparticle spectrum and flavour changing phenomena.* In the MSSM the universal soft SUSY breaking⁷⁸ guarantees natural suppression of the FCNC phenomena mediated by the sparticles. However, in the context of SUSY GUT this becomes insufficient, since the impact of physics between the Planck and GUT scales can strongly violate the soft-terms universality^{87,88}.

Predictive SUSY GUT frameworks, generically based on the HFE mechanism, employ a rich fermion sector (F -fermions) above the GUT scale, and all these fermions typically have large $O(1)$ Yukawa couplings. Then the soft masses of different sfermions with the same standard charges will have different RG running down from the scale M_P , so that at the scale M_X where the heavy states decouple from the light ones, the masses of the latter are no more universal⁸⁹. As a result, at lower energies the sfermions can arrive being already substantially split between different families, which would give rise to the FCNC phenomena.

Let us consider, as an example, the case of the $SU(6)$ model discussed in previous section. The FCNC problem emerges e.g. due to the third term in eq. (65), even if it does not contribute the light fermion masses and in fact works only for rendering the extra $(5 + \bar{5})_i$ states superheavy. The Yukawa couplings $\Gamma_{ij} \sim 1$, unless they are degenerate, would cause different RG running of particle masses from M_P down to the $SU(6)$ symmetry breaking scale V_H . Then the soft masses e.g. of sleptons \tilde{e}_i^c , contained in the multiplets 15_i ($i = 1, 2, 3$), should be strongly split already at the scale V_H . The non-universality of the slepton masses would induce decays like $\mu \rightarrow e + \gamma$, EDM of the electron, etc.⁸⁸.

The flavour-changing problem is a challenge for the relations of the two (20). In particular, splitting between the sfermion masses of the first two families would cause very serious problems, since in this case the $\mu \rightarrow e + \gamma$ decay rate, the rates of the $K^0 - \bar{K}^0$ transition, etc. would strongly exceed the experimental bound.

Besides the disorientation and plastification introduced in particle physics by Dimopoulos et al.⁹⁰, an idea of the non-Abelian horizontal symmetry, say $SU(3)_H$, can

be a natural way for solving the problem. For example, the model of ref.⁵⁰ (see section 2) can naturally render the sfermion masses unsplit (at least within first two families) in spite the renormalization effects caused by the large Yukawa constants. Since in this model the horizontal $SU(2)_H$ subgroup between the first two families is broken below the scale M_X , soft masses of the latter will have the same RG evolution down to the scale M_X and thus will remain unsplit. They will not get strong splitting due to mixing effects with the heavy F -states $\bar{X} + X$ and $V + \bar{V}$ neither, since the mixing angles are small: of the order of corresponding Yukawa constants in MSSM, λ_c, λ_s , etc. As for the third family fermions, their soft masses in general should be split from the ones of first two generations, since $\lambda_t \sim 1$ and thus they are strongly mixed to the soft masses of the corresponding F -states. However, splitting between the third and the first two families of sfermions is not yet a problem⁸⁸.

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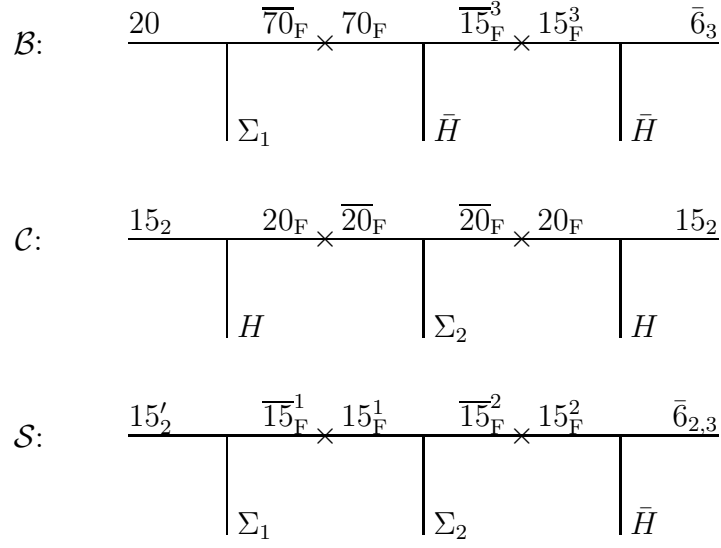


Figure 1: diagrams giving rise to the operators \mathcal{B} , \mathcal{C} , \mathcal{S}

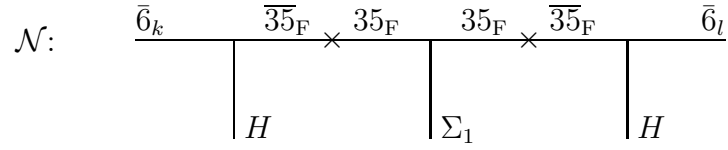


Figure 2: the diagram giving rise to the operator \mathcal{N} for neutrino mass

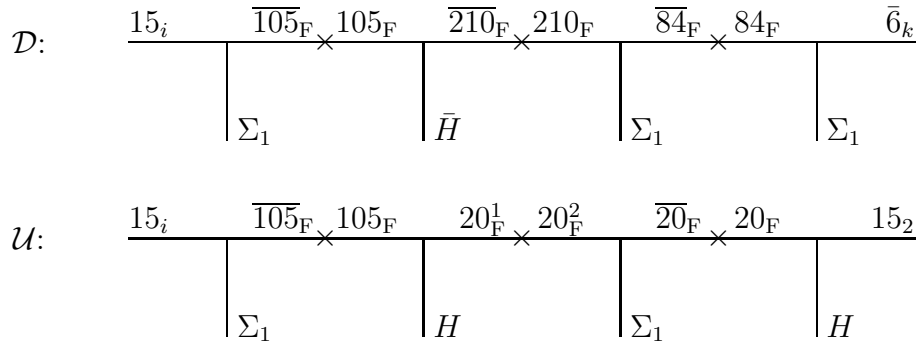


Figure 3: diagrams giving rise to the operators \mathcal{D} and \mathcal{U}